*International Research Journal of Innovations in Engineering and Technology (IRJIET)*

ISSN (online): 2581-3048

Volume 3, Issue 7, pp 17-21, July-2019

# Construction of a Number of Mathematical Curves by using Geometric Modeling - Apply to Machine Structures to Draw Continuous Curves without Interruption

**Manh Hong Do**

School of Mechanic, Descritive Geometry and Engineering Drawing Department, Hanoi University of Science and Technology, No.1, Dai Co Viet Street, Hai Ba Trung distric, Hanoi, Vietnam

***Abstract*** **- Curves with equations (EQT) are widely used to create surfaces of machine parts in machine manufacture. They make beautiful surfaces that appeal to consumers. These surfaces may be free surfaces or may be assembled surfaces. If the surfaces are assembled, they must be made accurately. For example, convolute of a circle is the profile of spur gears, as well as cycloid. There are some ways to draw curves. We can draw curves by using algebra or geometry. We also draw curves by using mechanical structures. If we use algebra to draw curves, we must determine the coordinates (COD) of the points of the curve and connect them according to its rules. The more points you should find, the more accurate the curve will be. Curve drawn by these methods is sporadic and consuming of time. Moreover, some curves still exist dead points. It is not easy to determine the COD of its. Whereas, if we use a geometric model to create structures to draw a curve, we will draw every point of the curve precisely and smoothly in a very short period of time. The author will analyze a simple geometric model that helps us create a structure that can draw many curves quickly and accurately.**

***Keywords:*** Equations, EQT Coordinates COD, Functions, FN.

## I. INTRODUCTION

Curves are widely used on objects' surfaces and machine parts. We have several methods of drawing curves. There are two methods in mathematics: geometric methods and algebraic methods. Drawing curves according to these methods is time-consuming and inaccurate. Because the curve is drawn through many different positions of a point. So the more positions of a point, the more accurate the curve will be. Determining the location of a point according to these methods is very discrete and complicated.

To draw curves fast and precisely, we must use a machine structure. When the machine structure moves, a point of it will go through all positions of it, including the special points of the curve. Previous machine structures only draw a curve. For example, elliptical drawing structure is only to draw ellipse.

Based on a mathematical model I would like to present a structure that can draw many smooth and precise curves.

## II. METHODS FOR DRAWING CỦRVES

### 2.1 Method 1: Algebraic method

To draw a curve according to this method, we will do the following steps:

- Making EQT of FN
- Determine the COD of the points by giving the values of the variable and replacing these values with the FN we get the values of the principle
- If the curve has some special points, we must survey to find them
- After that we will connect them according to the rules of the FN, we will draw the graph of the FN

Conclusion: The more points we determine, the more accurate the graph will be. This method is very sporadic and slow.

For example, we want to draw a circle, we must create its EQT.

The circle is the trajectory of a moving point M that is always a fixed distance R from a fixed point O. fixed point O is called the center point of the circle. The constant distance R is called the radius of the circle.

How to draw a circle:

- Making EQT of the circle
- To create the EQT of the circle we attach to its center O a COD system XOY. See fig 1
- Based on figure 2, we easily set the parametric EQT of the circle according to the rotation angle

$$X = Rcos\varphi$$
$$Y = Rsin\varphi \quad (1)$$

 

International Research Journal of Innovations in Engineering and Technology (IRJIET)

ISSN (online): 2581-3048

Volume 3, Issue 7, pp 17-21, July-2019

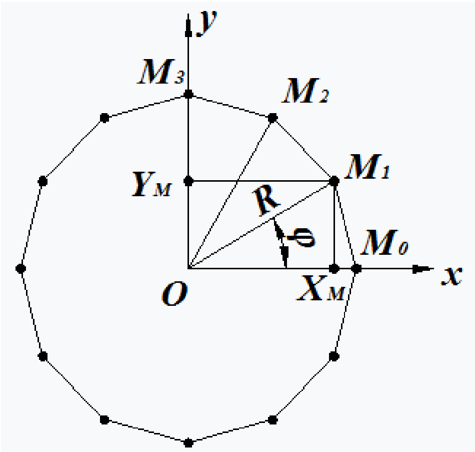


**Figure 1: How to Draw a Circle**

- Determine the COD of point M in different positions such as M1, M2, M3 ...
- We give the rotation angle the values as follows: $0^0$.$30^0$, $60^0$, $90^0$...
- Replacing the values of the rotation angle into the above EQT (1), we will calculate the corresponding values X and Y of point M. Therefore we will determine the positions of point M are: M0, M1, M2, M3...
- Connect these points such as M0, M1, M2, M3 ... together we will get the trajectory of M

Conclusion: The more positions of point M, the more accurate the curve will be. Determining the COD of special points of some curves such as extreme points, tangent points, limit points, dead points… is quite complicated. This method is very discrete and consuming of time.

### 2.2 Method 2: Geometric method

- Give a geometric model with constraints
- Let this model move
- Draw the trajectory of a point of the model

We can draw the trajectory of a point of the model as follows:

- When the model moves, base on each position of it, we can determine the new position of the point.
- Connect all the positions of the point according to the rule of motion of the model. That is an orbit of the point.

For example: How to draw involute of a circle

- Draw base circle.
- Divide the base circle into equal parts.
- Draw tangent lines to circle at dividing points.
- Determine the positions of point M on the tangent lines.
- Joining all positions of point M together, we will get a curve. This curve is the involute of the circle.

See figure 2:

- Divide the base circle into 6 equal parts
- Draw tangent lines to the circle at the points as 1,2,3,4,5,6
- Determine the positions of M on the tangent lines base on the following distances

$$\overline{2M_2} = \frac{\pi R}{3}$$

$$\overline{3M_3} = \frac{2\pi R}{3}$$

$$\overline{4M_4} = \frac{3\pi R}{3} = \boldsymbol{\pi R}$$

$$\overline{5M_5} = \frac{4\pi R}{3}$$

$$\overline{6M_6} = \frac{5\pi R}{3}$$

$$\overline{1M_7} = 2\boldsymbol{\pi R}$$

- Connect M1, M2, M3, M4, M5, M6, M7

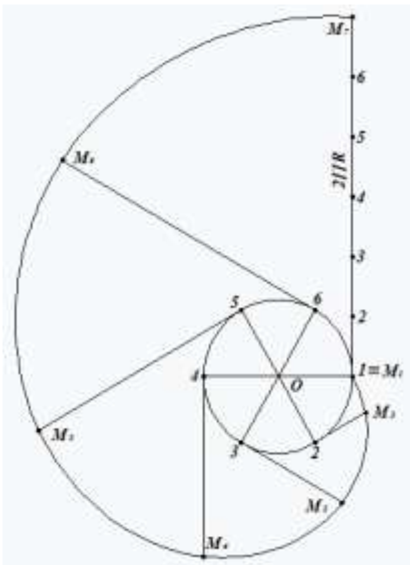


**Figure 2: How to Draw an Involute of a Circle**

Summary: This drawing is also sporadic and time- consuming.

### 2.3 Method 3: Draw curves by using mechanical structures

Previous structures often only draw one curve. For example, the elliptical structure is only used to draw ellipses.

 

*International Research Journal of Innovations in Engineering and Technology (IRJIET)*

**ISSN (online): 2581-3048**

**Volume 3, Issue 7, pp 17-21, July-2019**

The author of this paper will analyze a geometric model. Base on the following results we will create a model of a structure that can draw many curves

Let's solve a flat geometry problem as follows: (see fig3)

Given three points: point O, point A, point B on the plane P. Point O is the fixed point. Point O also is the origin of the COD. Point A and point B move. Point A rotates around point O, point B rotates around point A. Draw the trajectory of point B. OA = R, AB = r. when the rotation angle of point A equals to α then the rotation angle of point B equals to β.

We have: $\beta = \alpha + \alpha t \rightarrow$ (See fig 3)

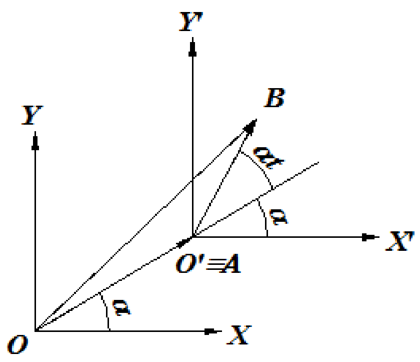


**Figure 3: A Flat Geometry**

Base on fig 3 we have:

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

We determine the COD of point B with the origin of COD is point O, so we have the following system of EQT:

$$x = R\cos\alpha + r\cos(\alpha + \alpha t)$$

$$y = R\sin\alpha + r\sin(\alpha + \alpha t) \quad (2)$$

The trajectory of point B will depend on t, R, r

We consider some of the following cases.

**2.2.1 Case 1** if t<0 then both point A and point B will turn in the opposite direction

- t = -1 then system of EQT (2) is:

$$x = R\cos\alpha + r\cos(\alpha - \alpha) = R\cos\alpha + r \rightarrow x - r = R\cos\alpha$$

$$y = R\sin\alpha + r\sin(\alpha - \alpha) = R\sin\alpha \rightarrow y = R\sin\alpha$$

$$\rightarrow (x - r)^2 + y^2 = R^2$$

This is the EQT of the circle. Its radius equal to R.

- t = -2 then system of EQT (2) is:

$$x = R\cos\alpha + r\cos(\alpha - 2\alpha) = (R + r)\cos\alpha$$
$$y = R\sin\alpha + r\sin(\alpha - 2\alpha) = (R - r)\sin\alpha$$

$$\rightarrow \left(\frac{x}{R+r}\right)^2 + \left(\frac{y}{R-r}\right)^2 = 1$$

This is the EQT of the ellipse. The major of it equal to R+r and the minor of it equal to R-r. The center point of it is point O. See fig 3

Special case: if R=r then system of EQT (2) is:

$$x = 2R\cos\alpha\ y = 0$$

This is a line AB. AB=4R. Both point A and point B are in line x. OA=OB=2R

- t = -3 then the system of EQT (2) is:

$$x = R\cos\alpha + r\cos(\alpha - 3\alpha) = R\cos\alpha + r\cos 2\alpha$$
$$y = R\sin\alpha + r\sin(\alpha - 3\alpha) = R\sin\alpha - r\sin 2\alpha$$

Depending on the two values of R and r we obtain different curves.

For example, if R = r then the system of EQT (2) is:

$$\rho = 2Rcos\frac{3\alpha}{2}$$

This is the EQT of lemlixcat with three wings. Its general form is:

$$\rho = A\cos 3\varphi$$

- t = -4 then the system of EQT (2)is:

$$x = R\cos\alpha + r\cos(\alpha - 3\alpha) = R\cos\alpha + r\cos 2\alpha$$
$$y = R\sin\alpha + r\sin(\alpha - 3\alpha) = R\sin\alpha - r\sin 2\alpha$$

This is the EQT of Astroide. Its general form is:

$$\rho^2 = A\cos 4\varphi + B$$

**2.2.2 Case 2**: t>0 then both points A and point B turn in the same direction. We will study some cases

- t = 1 then the system of EQT (2)is:

$$\rho^2 = R^2 = r^2 + 2rR.\cos\alpha$$

Here is the EQT of series curve depends on 2 values of R and r.

Especial case If R = r then the system of EQT (2) is:

$$\rho^2 = R^2.\cos\alpha + \frac{5}{4}R^2$$

This is the EQT of cardioide. General EQT of it is:

$$\rho^2 = A.\cos\alpha + \mathrm{B}$$

- t=2 we have a general EQT:

$$\rho^2 = A.\cos 2\alpha + \mathrm{B}$$

ISSN (online): 2581-3048

Volume 3, Issue 7, pp 17-21, July-2019

This is the EQT of a family of curves. It depends on the values of R and r.

Some value of t, the r / R ratio and the number of curves.

## III. RESULT AND DISCUSSION

**TABLE 1**

**Some results of some curves correspond to the values of t and the ratio R / r**

| Value of t | r/R ratio | Types of curves |
|---|---|---|
| -1 | <1 | Circle |
| -2 | <1 | Ellipse |
| -2 | 1 | Line |
| 1 | 2 | Cacdiot |
| $t = \frac{R-r}{r}$ | <1 | Epicycloide |
| $t = \frac{R+r}{r}$ | >2 | Hypocycloide |
| t = n | **>>1** | Astroide curve has n branches |
| t = -n | **1** | Lemlixcat curve has n wings |
| Ect… | | |

Based on the results above, we can create a machine structure with a speed box and a small motor to determine different factors of t. Combined with a rotary directional switch for t> 0 or t <0. The rotation motion from bar 1 to bar 2 with the selected R / r ratio gives different curves. See fig 4.

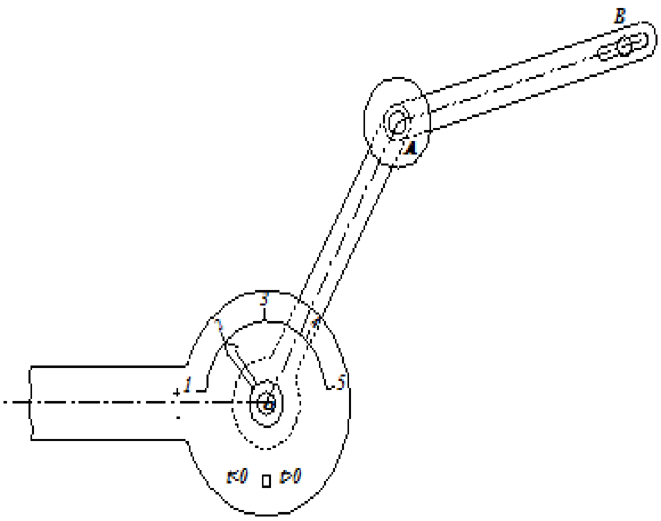


**Figure 4: A Machine Structure to Draw Curves**

When the mechanical structure move, the curve is drawn and connected through all points automatically without having to set up the EQT and finding its points.

Thanks to the motion mechanism, it creates a curve quickly and accurately.

To draw a new curve we just need to adjust the parameters as shown in Table 1.

## IV. CONCLUSION

We have some methods to draw a curve such as algebra, geometry and machine structure. But drawing curves by machine structure is the fastest and most accurate method. Previous structures could only draw a curve. But the machine structure above can draw many different curves accurately and smoothly.

## ACKNOWLEDGEMENT

In this paper, the author has studied and presented a structure that can draw many curves that previous structures could only draw a curve. In the future, I hope there will be many types of machines that can make complex profiles to create beautiful surfaces for machines and objects.

Today the world has some countries with population are getting older. So they need a lot of smart robots that can work correctly. Base on machine structures, we can create robot arms with precise orbits to serve human life.

## REFERENCES

[1] Dinh Tri Nguyen (editor-in-chief), Advanced Mathematics, volume 1, Education Publishing House 2006. Nguyen Thuy Thanh, Background of complex variable theory, VNU Publishing House, 2006.

[2] Dinh Tri Nguyen (editor) - Van Dinh Ta - Ho Quynh Nguyen ... Advanced Math Exercises Episode 1 - Dinh Tri Nguyen June 3, 2018In "Curriculum".

[3] Mechanics Program (Episode 2: Dynamics) – Sanh Do (Editor). September 2016, publisher of Hanoi University of Science and Technology.

[4] Dinh Gia Tuong, Ta Khanh Lam - Machine Principles Curriculum Episode 1. Nov 2014, publisher of Hanoi University of Science and Technology.

[5] Dinh Gia Tuong, Ta Khanh Lam - Machine Principles Curriculum Episode 2. Nov 2014 publisher of Hanoi University of Science and Technology.

[6] C F Cheung, L B Kong, W B Lee, S To, Modeling and simulation of freeform surface generation in ultra-precision raster milling. Department of Industrial and Systems Engineering, the Hong Kong Polytechnic University, Hong Kong, People's Republic of China.

ISSN (online): 2581-3048

Volume 3, Issue 7, pp 17-21, July-2019

[7] Tran The Luc, Trinh Minh Tu, Banh Tien Long, Surface forming textbook, September 28, 2017, publisher of Hanoi University of Science and Technology.

## AUTHOR'S BIOGRAPHY

**Manh Hong Do** M. E (1968), Degrees in mechanical engineering from Ha Noi University of Science and technology.

He is a Mater, Department of Mechanical Engineering, Ha Noi University of Science and Technology.

**Citation of this Article:**

Manh Hong Do, "Construction of a Number of Mathematical Curves by using Geometric Modeling - Apply to Machine Structures to Draw Continuous Curves without Interruption" Published in *International Research Journal of Innovations in Engineering and Technology (IRJIET)*, Volume 3, Issue 7, pp 17-21, July 2019.

*******