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Investigation of Short Term Deflection of Reinforced Concrete Flat Slabs

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***Abstract* - This research is dedicated to correct the Short Term of concrete Flat Slab deflection calculated based Branson equation used in both on ACI 318 code and ECB-207 code, Due to linear material behavior, sudden reduction of Effective inertia after cracking and 1D action of this equation(was conducted after studies in simply supported beams), calculated deflection from this equation is vastly overestimated form actual behavior, so a correction factor for deflection calculated From Branson equation is the main objective for this research. After verification of ABAQUS Software as FEM Simulation of Flat slab behavior, a parametric study is conducted to Make a charts for the correction Factor of deflection calculated using Branson equation. Assuming that Fc'=30mpa And Fy=400mpa, A parametric study is made on total of 75 Square simply supported Flat slab, Those slabs are modeled using both SAFE (using ACI-318 Cracking Criteria) And ABAQUS (Using CPDM Behavior for Concrete model) to Present Load Deflection Difference of Each case on each model, then Normalized Deflection (Δcracked/ΔABAQUS) is calculated for each λ (Mact/Mcr) to represent the different ratios of corrections over variance of super imposed dead load on slab. Finally 15 charts were presented for different cases of lower& top reinforcement ratios to correct the deflection calculated From CSISAFE (Using ACI318 Equation and Cracking Criteria).**

Keywords: Nonlinear analysis, Material nonlinearity, Reinforced concrete, reinforcing steel, Geometric nonlinearity, CPDM.

I. INTRODUCTION

The calculation of deflection and slope angle of concrete elements is essential for the element design and determine either If it's SAFE, and meet the structure serviceability or not, and it has many formula in different design code to determine it, most of them are using the concept of effective moment of inertia to determine the deflection either using manual or Finite element method, which is great, but back origin of these formulas, its derived mainly after testing of simple or continues reinforced concrete beams, which means the 2D effect is not considered, also, these formulas assume the linear behavior of concrete and only takes the effect of decreasing of cross section's rigidity.

II. HISTORICAL BACKGROUND

In 1963,Branson[1]Was the first to predict the formulation to calculate the short and long term deflection on reinforced concrete beams using the concept of effective moment of inertia for RC sections, this formula is used by many code e.g.:- ACI 318-05[2],the formula is expressed as follows:

$$I_{eff} = \left(\left(\frac{M_{cr}}{M_a}\right)^m\right) * I_g + \left(\left(1-\left(\frac{M_{cr}}{M_a}\right)\right)^m\right) * I_{cr} \quad (1)$$

Where Mcr is the cracked moment, and Ma is the applied moment, Igr is the gross moment of inertia, and Icr is the cracked moment of inertia, cracking moment is defined as Following

$$M_{cr} = \frac{(f_r)I_g}{Y_t} \quad (2)$$

While Fr is tensile strength of concrete is defined as:-

$$f_r = 0.62 * \sqrt{Fc'} \quad (3)$$

And Ma is for flat slab defined as the resultant moment of M11, M22 & M12 as Following:-

$$M_a = \sqrt{(M_{11})^2 + (M_{22})^2 + (M_{12})^2} \quad (4)$$

The parameter (m) was taken equal to 3. In the ACI Code and Most of codes, But Al-Shaikh and Al-Zaid [3]found out that the value of (m) decreases as the reinforcement ratio (ρ) of a concrete beam increases. Accordingly, they proposed the following equation for (m):

$$m = 3 - 0.8\rho \quad (5)$$

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After That, in 1991 , Al-Shaikh and Al-Zaid[3]derived a formula based on testing results of beam with 200mm square section with 0.4% of the balanced reinforcement, the formula is expressed as follows:

$$I_{eff} = \left(\left(\frac{L_{cr}}{L}\right)^m\right) * I_g + \left(\left(1-\left(\frac{L_{cr}}{L}\right)\right)^m\right) * I_{cr} \quad (6)$$

Where Lcr is the cracked Length and L is the un-cracked Length and (m) value is expressed in equation 2

In 2005 Bischoff [4] proposed a modified approach of Branson's Equation after Several Tests To Attempt a convergence to them ,The Equation was proposed as follow:-

$$I_{eff} = \frac{I_{cr}}{1-\left(\left(\frac{M_{cr}}{M_a}\right)^m\right) * \left(1-\left(\frac{I_{cr}}{I_g}\right)\right)} \quad (7) \qquad Where\ m = 2$$

And in 2012,S. Roohollah Mousavi [5], he managed , after several trail, an algorithm to correct Branson's equation taking into consideration the GFRP effect, he proposed a model of equation for beams with GFRP RFT as the Following:-

$$I_{eff} = \left(0.13 * \left(\frac{M_{cr}}{M_a}\right)^m\right) * I_g + \left(0.89 * \left(1-\left(\frac{M_{cr}}{M_a}\right)\right)^m\right) * I_{cr} \quad (8)$$

Where

$$m = -0.24 * \left(\frac{\rho}{\rho_b}\right) + 5.35 * \left(\frac{M_{cr}}{M_a}\right) + 2.28 * \left(\frac{E_f}{E_s}\right) \quad (9)$$

He said that the proposed equations can better predict deflection when effective moment of inertia is less than Icr, especially at high levels of loading and reinforcement ratios.

In 2014 Ayman Embaby[6] proposed the equation Based on secant stiffness model to formulate the equations below:-

$$EI_{eff} = \frac{M_{av}}{\Phi_{eff}} \quad (10)$$

Where

$$\Phi_{eff} = \Phi_{cr} * \left(\frac{\Phi_y - \Phi_{cr}}{M_y - M_{cr}}\right) * (M_a - M_{cr}) \quad (11)$$

$$\Phi_{cr} = \frac{e_r}{z} = \frac{\frac{f_r}{E}}{z} \quad , \quad \Phi_y = \frac{0.002}{d-\left(\frac{A_s * f_y}{0.7 * f_c * b}\right)} \quad (12)$$

$$M_{av} = 0.5 * (M_a + M_{cr}) \quad (13)$$

$$M_y = 0.87 * A_s * f_y * d \quad (14)$$

And compare between his model and all previous models in a study using strip method on simply supported beams of 10m and 300mm*1000mm cross section with RFT ratios of 1.0%, 1.5%, and 2.0%, the study shows that his formula gives conservative, yet more accurate results

Finally in 2015, PATEL, K.A.[7]used ABAQUS to verify test data form simply supported beam with 2.5 m clear span and Square section of 200*200 mm with bottom RFT Asb=201.12mm2 and Top RFT Ast=78.54mm2, beam has been modelled using B21 elements (2-node linear Timoshenko beam element). Under service load, for material model, he used concrete smeared cracking constitutive model to model the concrete behavior and plastic model to model the steel rft behavior, close agreement is observed between the results From FEM and experiments,

After His parametric study, He proposed the explicit formula as follows:

$$I_{eff} = \left(\frac{3 * I_g}{1+e^{-\left(-7.4688+\left(\frac{8.7116}{1+e^{-H1}}\right)+\left(\frac{0.3754}{1+e^{-H2}}\right)+\left(\frac{11.6985}{1+e^{-H3}}\right)+\left(\frac{10.7167}{1+e^{-H4}}\right)+\left(\frac{0.6177}{1+e^{-H5}}\right)+\left(\frac{22.9397}{1+e^{-H6}}\right)\right)}}\right) \quad (15)$$

Where H1~H6 are factors depends on Mcr, Mact, Ig, Ieff,&rt

Then he validate the equation by, simulating six t-sections simple beams with different dimensions and bottom and top reinforcements values and conditions, the mid-span deflections obtained from the proposed explicit expression and FEM are close for the range of the load considered. The difference between FEM and proposed explicit expression is 2.91% as compared to 16.81% difference between FEM and ACI 318 [2]at 4 Wcr, (Wcr = cracking uniformly distributed load).

For continuous beam problem he simulate 2 spans T-Section continuous beam and he obtain the difference between FEM and proposed explicit expression is 5.34% as compared to 28.25% difference between FEM and ACI 318 [2]at 5 Wcr.

III. FINITE ELEMENT ANALYSIS

For both simulation of actual behavior of slabs using ABAQUS FEM, and Branson equation behavior using CSISAFE fem, both geometrical and material model is assigned for both software's as following:

a) For ABAQUS CAE

i) Material Model

For Concrete Model, we choose to use concrete damage plasticity model to represent concrete behavior, concrete damage plasticity model is a constitutive model is based on combining 2 main approaches ,elastic damaged model and elastic plastic law to present a new model that can capture the

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constitutive behavior of experimental loading unloading for concrete in both tension and compression either in dynamic or monotonic loading[8].

According to [9], The damage plasticity constitutive model is based on the following stress-strain relationship:

$$\sigma = (1 - \omega_t)\bar{\sigma}_t + (1 - \omega c)\bar{\sigma}_c \qquad (13)$$

Where $\bar{\sigma}$t and $\bar{\sigma}$c are the positive and negative parts of the effective stress tensor r, respectively, and ωt and ωc are two scalars damage variables, ranging from 0 (undamaged) to 1 (fully damaged). The effective stress Tensor $\bar{\sigma}$ is defined by the double dot product of stiffness tensor with elastic as Follow:

$$\bar{\sigma}t = D_e : (\varepsilon - \varepsilon_p) \qquad (14)$$

Where De is the elastic stiffness tensor based on the elastic Young's modulus E and Poisson's ratio ν, ε is the strain tensor and εp is the plastic strain tensor.

The plasticity model is based on the effective stress, which is independent of damage. The model is described by the yield function, the flow rule, the evolution law for the hardening variable, and the loading–unloading conditions.

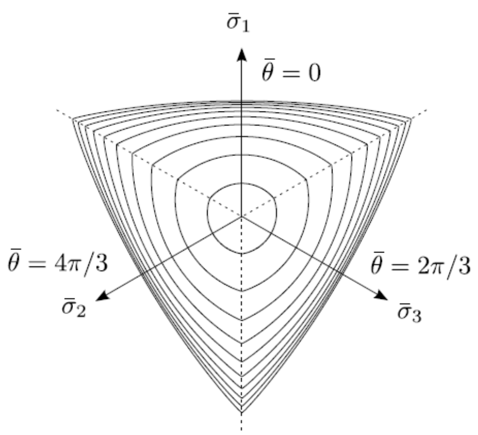

Figure 1: The Evolution of the Deviatoric Section of the Yield Surface during Hardening for A Constant Volumetric Stress of $\bar{\sigma}_V = -f_c/3$

For uniaxial Compression behavior, we used the Domingo J. Carreira and Kuang-Han Chu[10]Equation as Following:

$$\frac{fc}{fc'} = \frac{(\beta(\varepsilon - \varepsilon_c'))}{\beta - 1 + (\varepsilon - \varepsilon_c')^{\beta}} \qquad (15)$$

Where

$$\beta = \frac{1}{(1 - (\frac{fc'}{\varepsilon c' * E})} \qquad (16)$$

For uniaxial tension behavior, we used hsu[11] Equation as Following:

$$\sigma_t = f_t * \left(\frac{\varepsilon}{\varepsilon_{cr}}\right)^{.0.4} \qquad (17)$$

Where

$$f_t = 0.1 * Fc' \qquad (18)$$

For material plasticity implementation, coefficient values were taken based of ABAQUS manual recommendations, **Dilation angle ψ** were taken by 30 degrees, **Eccentricity (e)** were taken by 0.1, **equibiaxial to uniaxial compressive strength(F_{b0}/F_{c0})** were taken By 1.16, **Viscosity parameter(K)** Were taken by 0.025.

For Steel Reinforcement we choose elastic perfect plastic model to represent the steel behavior in both compression and tension behavior.

ii) Geometrical Model

Elements in ABAQUS are named as follows:

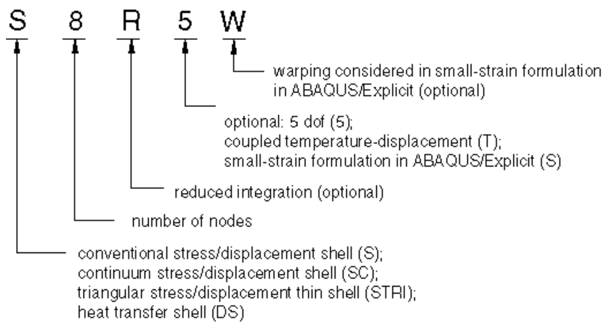

Figure 2: Naming Element Coding System in ABAQUS

So S8R is an 8-node, quadrilateral, first-order interpolation, stress/displacement shell element with reduced integration.

So According to [12] the general-purpose of conventional shell elements provide accurate solutions.

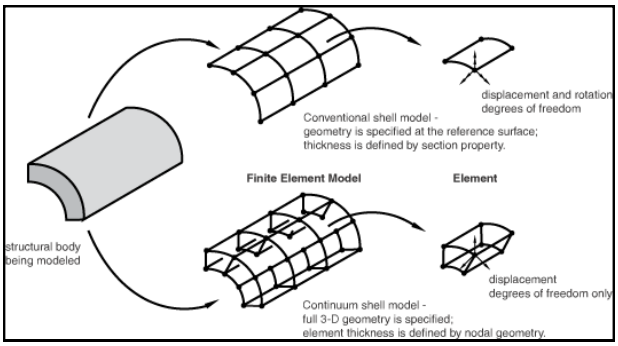

Figure 3: ABAQUS Conventional versus continuum shell element

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b) For CSISAFE

i) Material Model

CSI SAFE[13]assume Linear Elastic Perfect Plastic Behavior for Both Steel and Concrete.

ii) Geometrical Model

CSI SAFE[13]uses Shell Element with effective inertia calculation within Stiffens matrix Base on Branson Equation Mentioned on Equation 01.

IV. VERIFICATION

In order to calculate corrections for the ACI equation, a verification of total of 6 slabs were modeled Using both CSI SAFE (using ACI-318 Cracking Criteria) And ABAQUS CAE (Using CPDM Behavior for Concrete model) to present load deflection difference of each case on each model.

As shown in Figs (4, 5) results of ABAQUS FEM model are reliable for Parametric Study.

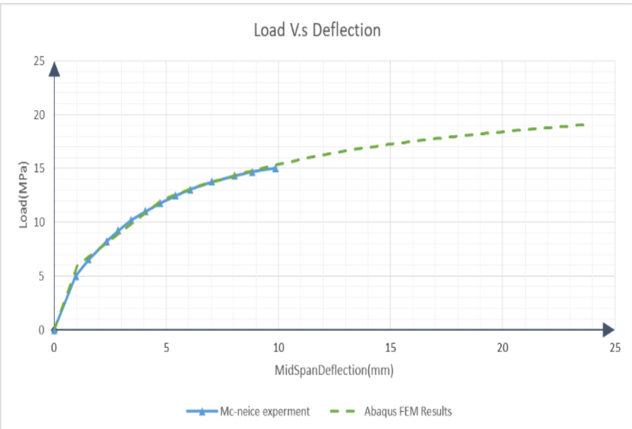

Figure 4: Mc Neice Load-Deflection Data for Both Experimental and FEM

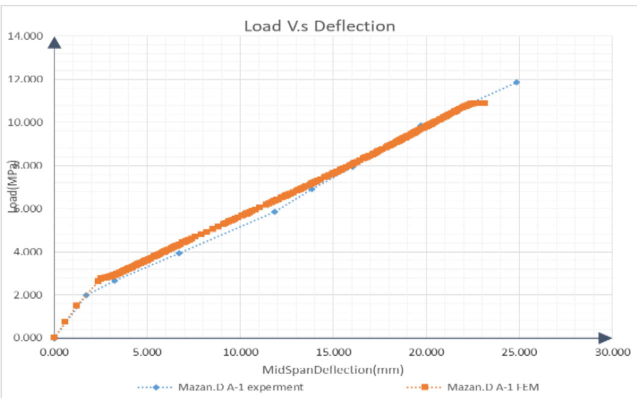

Figure 5: Mazan D Load-Deflection Data for Both Experimental and FEM

V. PARAMETRIC STUDY

A parametric study is done on Total of 75 rectangular simply supported Slabs (5 lengths of 6 to 10m with 1m step, 3 Bottom Reinforcement Ratios Percentages from 0.20% to 0.40% With 0.1% Step,5 Top Reinforcement Ratios Percentages From 0.00% to 0.40% With 0.1% Step) as Shown in Table01 e.g. (Ast=0.0%).

Table 1
Cases Names and Properties for Ast=0%

Case Name	Slab Geometry				Bottom RFT			Top RFT		
	L (mm)	R(L/Ls)	Ls (mm)	T (mm)	Sb	ϕb	Asb%	St	ϕt	Ast%
A-6000-100-02-00	6000	1.00	6000	185	212	10	0.2%	0	0	0.0%
A-6000-100-03-00	6000	1.00	6000	185	142	10	0.3%	0	0	0.0%
A-6000-100-04-00	6000	1.00	6000	185	106	10	0.4%	0	0	0.0%
A-7000-100-02-00	7000	1.00	7000	215	183	10	0.2%	0	0	0.0%
A-7000-100-03-00	7000	1.00	7000	215	122	10	0.3%	0	0	0.0%
A-7000-100-04-00	7000	1.00	7000	215	91	10	0.4%	0	0	0.0%
A-8000-100-02-00	8000	1.00	8000	245	160	10	0.2%	0	0	0.0%
A-8000-100-03-00	8000	1.00	8000	245	107	10	0.3%	0	0	0.0%
A-8000-100-04-00	8000	1.00	8000	245	80	10	0.4%	0	0	0.0%
A-9000-100-02-00	9000	1.00	9000	275	143	10	0.2%	0	0	0.0%
A-9000-100-03-00	9000	1.00	9000	275	95	10	0.3%	0	0	0.0%
A-9000-100-04-00	9000	1.00	9000	275	71	10	0.4%	0	0	0.0%
A-10000-100-02-00	10000	1.00	10000	305	129	10	0.2%	0	0	0.0%
A-10000-100-03-00	10000	1.00	10000	305	86	10	0.3%	0	0	0.0%
A-10000-100-04-00	10000	1.00	10000	305	64	10	0.4%	0	0	0.0%

Those slabs are modeled using both SAFE (using ACI-318 Cracking Criteria) and ABAQUS (using CPDM behavior for concrete model) to present load deflection difference of each case on each model.

VI. RESULTS AND DISCUSSION

As Shown in Table (2) and Fig (6) for Example of A-6000-100-02-00 the difference between Working Loading Causing the Deflection Limitation (L/250) in SAFE Model was 5.87KN/m2 while in ABAQUS model it was 7.82KN/m2, **So SAFE Model gives 22% less Load than ABAQUS.**

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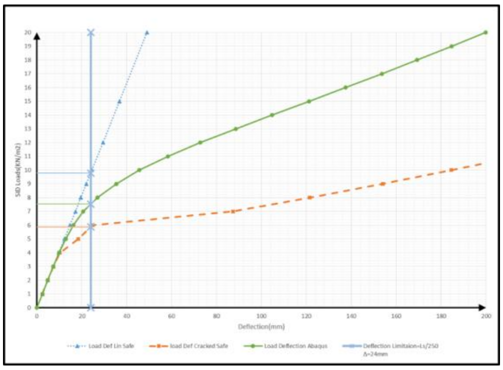

Figure 6: Load Deflection for Case A-6000-100-02-00

TABLE 2
Case A-6000-100-02-00 Limitation load for Code Limitation (l/250) For ABAQUS, CSI SAFE Cracking Criteria & linear Calculation

Case	Limitation Load (KN/m2)	λ	Diff
Abaqus	7.53	2.14	0%
Linear Case	9.79	2.78	30%
Cracked Safe Case	5.87	1.67	-22%

For correction procedure of deflection calculated from CSI SAFE (Using ACI Cracking Criteria), relation between long span and normalized deflection for each actual to cracking moment ratios (λ), is presented for 75 slab throughout 15 chart, each chart present this relation for specific bottom and top reinforcement ratio, 3 of them are shown in figs (7, 8 & 9) [(Asb=0.2%, 0.3% & 0.4%) Respectively & Ast (=0.0%)]

For example, in flat slab cases A-6000-100-02-00,A-6000-100-03-00,A-6000-100-04-00, for material properties shown in table (3), and at super imposed dead load W=5.10 kN/m2, staining actions and creaking deflection are extracted from CSISAFE as shown in table (4,5), then after calculating cracking moment [Eq. (2)], Actual Moment[Eq. (3)], and actual to cracking moment ratios **λ=Mact/Mcr**, then going to correction charts for each specific bottom and top reinforcement ratio (Fig(7.8,9)), normalized deflection is extracted for each case for each case ,therefore correction factor is defended as the inverse of normalized deflection as shown in table (05)

TABLE 3
Material Properties for Square Flat Slab L=6000 mm

Properties	Value(Mpa)	Unit
L	6000.0	m
T	245.0	mm
Fy	400.0	Mpa
Fc'	30.0	Mpa
Fr	3.4	Mpa
Mcr	19.5	(kN.m\m')

TABLE 4
Straining Actions

Case Name	Bottom RFT	Top RFT	Moment Component And Resultant(kN.m\m')				
	Asb%	Ast%	W(kN/m2)	M11	M22	M12	Mact
A-6000-100-02-00	0.2%	0.0%	5.10	19.95	19.95	0.00	28.21
A-6000-100-03-00	0.3%	0.0%	5.10	19.95	19.95	0.00	28.21
A-6000-100-04-00	0.4%	0.0%	5.10	19.95	19.95	0.00	28.21

TABLE 5
Cases Cracked Deflection and Correction Factors

Case Name	Deflection	λ=(Mact/Mcr)	Correction Chart Name	Normalized Deflection	Correction Factor
	ΔCracked(mm)	La			
A-6000-100-02-00	18.48	1.45	A-02-00	1.50	0.67
A-6000-100-03-00	17.88	1.45	A-03-00	1.48	0.68
A-6000-100-04-00	16.60	1.45	A-04-00	1.42	0.71

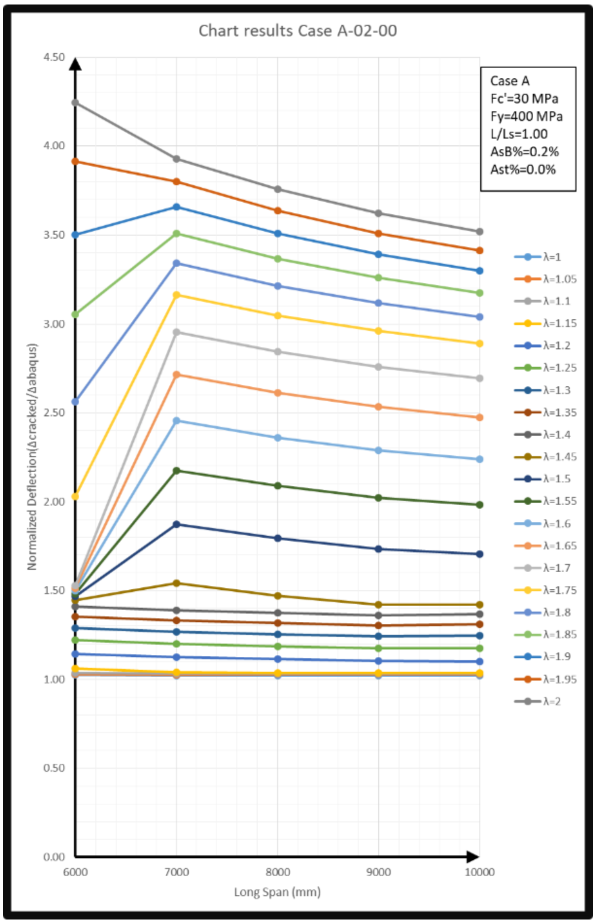

Figure 7: Span Normalized Deflection Curve for Case A-02-00

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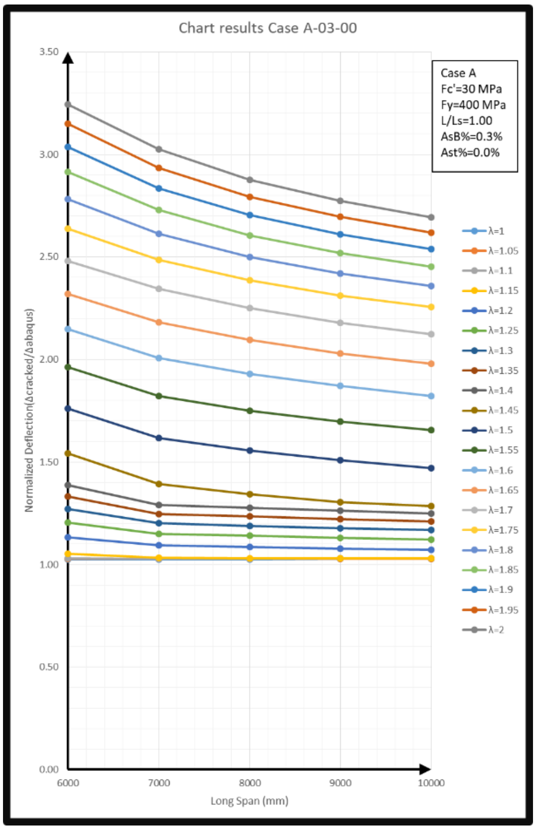

Figure 8: Span Normalized Deflection Curve for Case A-03-00

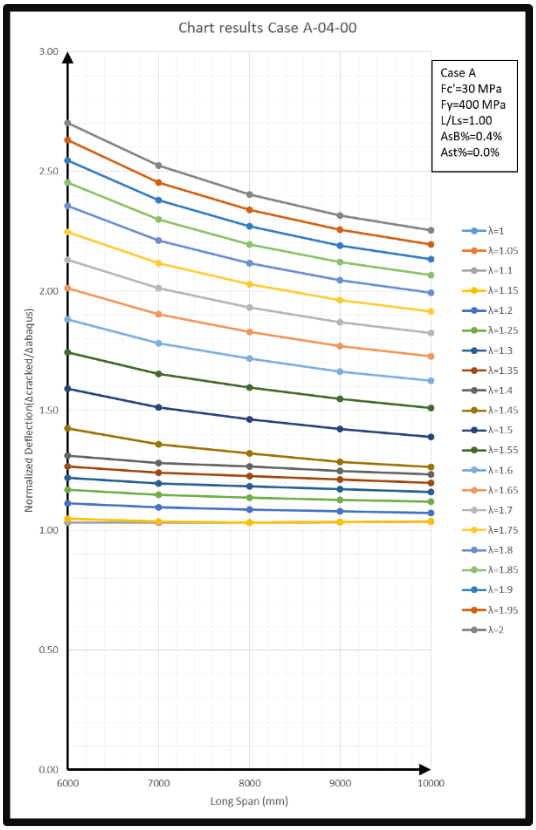

Figure 9: Span Normalized Deflection Curve for Case A-04-00

VII. CONCLUSIONS

This study presents correction for CSI safe short term deflection (based on Branson equation used in ACI318-14) though out correction charts for specific:

1) Lower Reinforcement Ratio
2) Top Reinforcement Ratio
3) Characteristic strength of concrete
4) Yield strength of Steel

Those charts are relations between long span of square simply supported slab verses normalized deflection (Δcracked/ΔABAQUS) for various λ= (Mact/Mcr) values, as the correction factor is the inverse of the normalized deflection extracted from those charts.

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