

Design of Interval Type-2 Fuzzy Logic Controllers Based on Several Evolutionary Optimization Algorithms for Inverted Pendulum System

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Abstract - This paper compared the performance between type-1 and type-2 of fuzzy logic controller for inverted pendulum system as a controlling plant. The parameters of each controller were tuned with four of evolutionary optimization algorithms (Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), and Social Spider Optimization (SSO)). The comparisons carried out between the controllers of fuzzy type-1 and fuzzy type-2 as well as between several optimization algorithms with fuzzy type-2 itself. The results of comparisons proved that the fuzzy type-2 controller with SSO has been achieved the best time response characteristics and the least tuning time.

Keywords: Fuzzy Logic Controllers, Optimization Algorithms, Inverted Pendulum System, Genetic Algorithm, Particle Swarm Optimization, Ant Colony Optimization, Social Spider Optimization.

I. INTRODUCTION

The inverted pendulum is an excellent system to verify the capability of controllers in control engineering field, and a very good test benchmark for various complicated control searching problems. It is a non-linear dynamic system with a single input (force applied to cart) and double output (the cart position and pendulum angle), thus inverted pendulum is single-input multi output (SIMO) system.

Inverted pendulum has common used and many application as launching rockets and missiles guidance [1], Commercial application of inverted pendulum model is the two wheel scooter (Segway) [1], Humanoid robots that walking in upright way is an application of inverted pendulum model [1].

Some researchers neglect the friction in the mathematical model of inverted pendulum for linearize the system [2-4] it is not legal approximation because physically the cart and the pendulum pole encounter some friction. The researchers of [5,6] provided clear steps in mathematical modeling to the

system using Euler-Lagrange's equations to present the equations of motion.

There are many different controllers algorithms and design techniques for stabilization of cart position and pendulum angle in inverted pendulum controlling such as Proportional Integral Derivative (PID) controller [10], Fuzzy logic inference (FLI) controller [2,8,9], Fractional Order PID (FOPID) controller [7]. The fuzzy logic inference are widely used to control an inverted pendulum [8, 9]. In [11-13] the fuzzy type-1 and fuzzy type-2 controllers combined with PID to product fuzzy like PID controller.

Many criticisms were raised against fuzzy type-1 regarding its incomplete capability to analyze considerable uncertainty, which contradicts the notion of incertitude denoted by the word 'fuzzy'. So Professor Zadeh published another paper included a second version of the fuzzy set [3], called Fuzzy Type-2.

The type-2 fuzzy logic system and controller applications have a big attention in the researches at last few years. Mendel et al. Proposed the interval type-2 IT2 fuzzy logic system in [25]. Wu Dongrui presents a mathematical and programming illustrating of IT2 fuzzy logic system [26].

The tuning of controller parameters is very important matter; it can affect the system stability as the controller type. Hence, selection the best parameters is also another target. The first method is the try and error as in [14]. This way consumes big effort and time therefore, many evolutionary optimization algorithms are widely common used as: as Particle Swarm Optimization (PSO), Ant colony Optimization (ACO), social spider optimization (SSO), Genetic Algorithm (GA), Bees Algorithm.

The paper [15] comparing both Genetic Algorithm (GA) and Bees Algorithm in tuning PID parameters. In [16, 17] Genetic Algorithm are used to find controller parameters but searcher of [18] used Bees Algorithm to get best selection for parameters. Zhe Sun et al. compare Type-1 and type-2 of FLS

for double inverted pendulum with optimizes them by genetic algorithm.

The social spider optimization (SSO) is very rarely used to find the parameters with controllers of Inverted pendulum even with its superiority compared with other artificial intelligent algorithm as was proven in the results of this paper.

This paper implements a fuzzy-PID controller and its parameters tuning by several evolutionary optimization algorithms. This research subject will be extensively modify and applied in several engineering domains.

After its introduction, the rest of the paper is organized as follows, Sections 2 presents the mathematical model of the

inverted pendulum system, Section 3 presents a brief explain of Fuzzy type-2 controllers, Sections 4 presents a controller design, Section 5 presents the types of Optimizations with Evolutionary Algorithms, Section 6 presents the results and discussions Section , Section 7 presents conclusions.

II. MATHEMATICAL MODEL

The mathematical model of the inverted pendulum system will derived in this paper according to the Lagrange motion equations of second type [5]. Lag range equations are the most common way adopted the mechanical analytical method to deriving the system equation of motion for the complex systems.

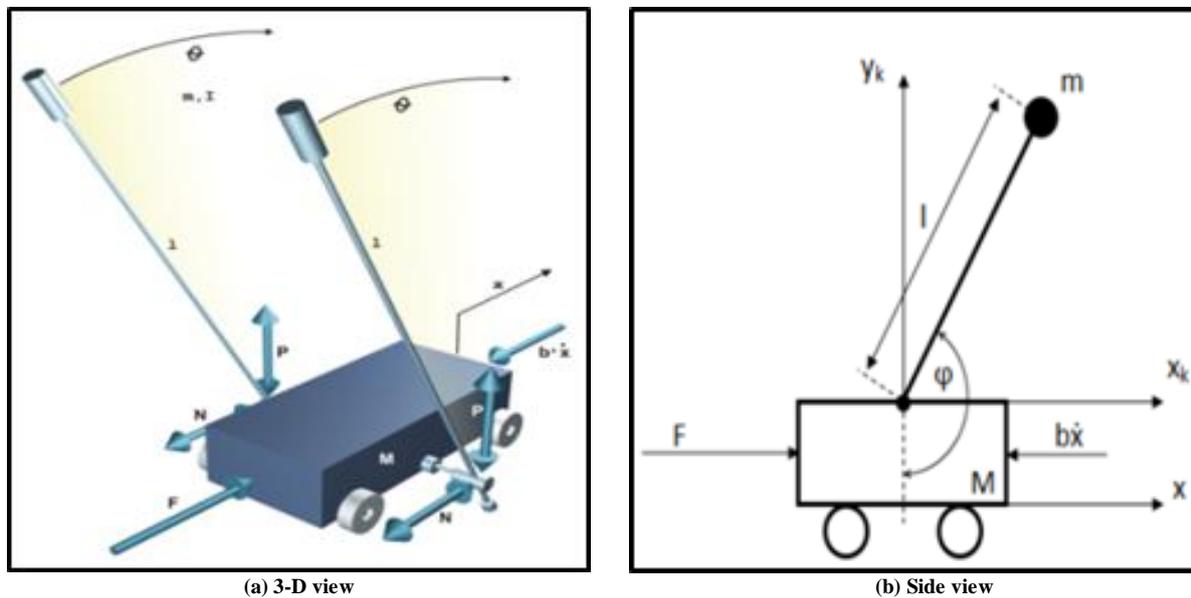


Figure 1: Inverted Pendulum cart

The parameters of mathematical model for inverted pendulum system are listed in Table 1. The values of the parameters represented the physical values of digital pendulum control instrument experiments system 33-936S who's used in real time implementation of research.

TABLE 1
physical parameters description of pendulum system

Parameter	Symbol	Value	Unit
Mass of cart	M	2.4	Kg
Mass of pendulum	m	0.23	Kg
Length of pendulum	l	0.36	m
Coefficient of friction for the cart	b_1	0.05	$Nm^{-1}s^{-1}$
Coefficient of friction for the pendulum	b_2	0.005	$Nrad^{-1}s^{-1}$
Mass moment of inertia of the pendulum	I	0.099	Kg /m^2
	g	9.81	m/s^2

Gravitation force	F	-	N
Force applied to the cart	X	-	m
Position of the cart	θ	-	rad

The kinetic energy law is

$$(KE) = \frac{1}{2} MV^2 \quad (1)$$

X = Position of cart

Derivation of position X by time is \dot{X}

\dot{X} = velocity of cart

The kinetic energy of inverted pendulum is

$$E_{KV} = \frac{1}{2} M\dot{X}^2 \quad (2)$$

X_K = Horizontal position coordinates of pendulum

Y_K = Vertical position coordinates of pendulum

$$X_K = X + l \sin \theta \quad (3)$$

$$Y_K = l \cos \theta \sin \theta \quad (4)$$

Derivation of positions (X_K, Y_K) by time is velocities (V_{KX}, Y_{KY})

$$V_{KX} = \dot{X} + l \cos \theta \sin \theta \quad (5)$$

$$V_{KY} = -l\dot{\theta} \cos \theta \sin \theta \quad (6)$$

The velocity square for pendulum will be

$$|V_K^2| = V_{KX}^2 + V_{KY}^2 \sin \theta \quad (7)$$

$$= \dot{X}^2 + 2l\dot{X}\dot{\theta} \cos(\theta) + l^2\dot{\theta}^2(\cos(\theta))^2 + l^2\dot{\theta}^2(\sin(\theta))^2$$

$$= \dot{X}^2 + 2l\dot{X}\dot{\theta} \cos(\theta) + l^2\dot{\theta}^2((\cos(\theta))^2 + (\sin(\theta))^2)$$

$$= \dot{X}^2 + 2l\dot{X}\dot{\theta} \cos(\theta) + l^2\dot{\theta}^2$$

Since the kinetic energy of pendulum is

$$E_{KK} = \frac{1}{2} mV^2$$

$$\therefore E_{KK} = \frac{1}{2} m\dot{X}^2$$

$$= \frac{1}{2} m(\dot{X}^2 + 2l\dot{X}\dot{\theta} \cos \theta + l^2\dot{\theta}^2) \quad (8)$$

$$E_{KV} = \frac{1}{2} M\dot{X}^2 + \frac{1}{2} I\dot{\theta}^2 \quad (9)$$

E_K is the kinetic-energy for the system

$$E_K = E_{KK} + E_{KV} \quad (10)$$

$$E_K = \frac{1}{2} m(\dot{X}^2 + 2l\dot{X}\dot{\theta} \cos \theta + l^2\dot{\theta}^2) + \left(\frac{1}{2} M\dot{X}^2 + \frac{1}{2} I\dot{\theta}^2\right)$$

∴ Totally Kinetic-energy for system after simplify is

$$E_K = \frac{1}{2} (M + m)\dot{X}^2 + \frac{1}{2} (I + ml^2)\dot{\theta}^2 + ml\dot{X}\dot{\theta} \cos \theta$$

Lagrange eq. of velocity of cart (X) and angle of pendulum θ that describe the system motion of inverted pendulum are

$$\frac{d}{dt} \left(\frac{\partial E_K}{\partial \dot{X}} \right) - \frac{\partial E_K}{\partial X} = Q_X \quad (11)$$

$$\frac{d}{dt} \left(\frac{\partial E_K}{\partial \dot{\theta}} \right) - \frac{\partial E_K}{\partial \theta} = Q_\theta \quad (12)$$

The terms of above equations must be calculated as follows

$$\frac{\partial E_K}{\partial \dot{X}} = (M + m)\dot{X} + ml\dot{\theta} \cos \theta \quad (13)$$

$$\frac{d}{dt} \left(\frac{\partial E_K}{\partial \dot{X}} \right) = \frac{d}{dt} [(M + m)\dot{X} + ml\dot{\theta} \cos \theta] \quad (14)$$

$$= (M + m) \frac{d}{dt} \dot{X} + ml \frac{d}{dt} (\cos \theta \dot{\theta})$$

$$= (M + m)\ddot{X} + ml(\cos(\theta)\ddot{\theta} - \dot{\theta}^2 \sin(\theta))$$

$$= (M + m)\ddot{X} + ml\ddot{\theta} \cos(\theta) - ml\dot{\theta}^2 \sin(\theta)$$

$$\frac{\partial E_K}{\partial X} = 0 \quad (15)$$

$$Q_X = F - b_1\dot{X} \quad (16)$$

∴ 1st Lagrange equations will become

$$[(M + m)\ddot{X} + ml\ddot{\theta} \cos(\theta) - ml\dot{\theta}^2 \sin(\theta)] - [0] = [F - b_1\dot{X}] \quad (17)$$

$$(M + m)\ddot{X} + ml\ddot{\theta} \cos(\theta) - ml\dot{\theta}^2 \sin(\theta) - F + b_1\dot{X} = 0$$

$$\ddot{X} = \frac{-ml\ddot{\theta} \cos(\theta)}{(M + m)} + \frac{ml\dot{\theta}^2 \sin(\theta)}{(M + m)} + \frac{F}{(M + m)} + \frac{b_1\dot{X}}{(M + m)} \quad (18)$$

And 2nd equation calculated by same way

$$\frac{\partial E_K}{\partial \dot{\theta}} = (I + ml^2)\dot{\theta} + ml\dot{X} \cos \theta \quad (19)$$

$$\frac{d}{dt} \left(\frac{\partial E_K}{\partial \dot{\theta}} \right) = \frac{d}{dt} [(I + ml^2)\dot{\theta} + ml\dot{X} \cos \theta] \quad (20)$$

$$= (I + ml^2) \frac{d}{dt} \dot{\theta} + ml \frac{d}{dt} (\cos \theta \dot{X})$$

$$= (I + ml^2) \ddot{\theta} + ml (\cos \theta \ddot{X} - \dot{X} \dot{\theta} \sin \theta)$$

$$= (I + ml^2) \ddot{\theta} + ml \cos \theta \ddot{X} - ml \dot{X} \dot{\theta} \sin \theta$$

$$\frac{\partial E_K}{\partial \theta} = -ml \dot{X} \dot{\theta} \sin \theta \tag{21}$$

$$Q_\theta = -mgl \sin \theta - b_2 \dot{\theta} \tag{22}$$

∴ 2nd Lagrange equations will become

$$[(I + ml^2) \ddot{\theta} + ml \cos \theta \ddot{X} - ml \dot{X} \dot{\theta} \sin \theta] - [-ml \dot{X} \dot{\theta} \sin \theta] = [-mgl \sin \theta - b_2 \dot{\theta}] \tag{23}$$

$$(I + ml^2) \ddot{\theta} + ml \cos \theta \ddot{X} + mgl \sin \theta + b_2 \dot{\theta} = 0$$

$$\ddot{\theta} = -\frac{ml \cos \theta \ddot{X}}{I + ml^2} - \frac{mgl \sin \theta}{I + ml^2} - \frac{b_2 \dot{\theta}}{I + ml^2} \tag{24}$$

III. FUZZY TYPE-2 LOGIC CONTROLLER

Fuzzy type-2 is an enhanced type of fuzzy type-1. The membership function MF of fuzzy type-1 has a single degree of membership value, so it does not achieve the uncertainty requirements in satisfactory way. Type-2 FIS has a range of values in degree of membership, so this type named (interval fuzzy type-2) IT2-FIS.

In interval type-2 the MF construct by two functions, upper MF (UMF) and lower MF (LMF). The (UMF) is the same as for the membership function in the fuzzy type-1. The (LMF) less than or equal the (UMF) for all UOD input values. The degree of membership belongs to a range between the LMF and UMF boundaries. The area between (UMF) and (LMF) called the footprint of uncertainty (FOU). The Figure (2) below shows the UMF, LMF and FOU for a type-2 triangular MF.

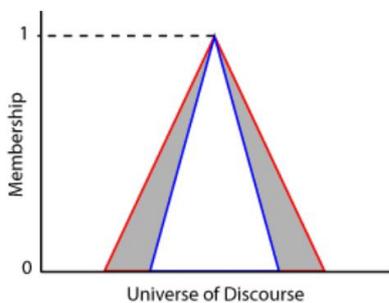


Figure 2: Upper and lower MF of IT2-FIS

The fuzzy inference system (FIS) includes five stages, Fuzzification, Application, Implication, Aggregation and Defuzzification. The defuzzification stage converts the single

type-2 fuzzy set to single crisp value. The conversion process require reduction the (type-2) fuzzy set to (type-1) fuzzy set, this process called (type reduction). The block diagram shown in Figure 3 represent the structure of a type-2 FLS.

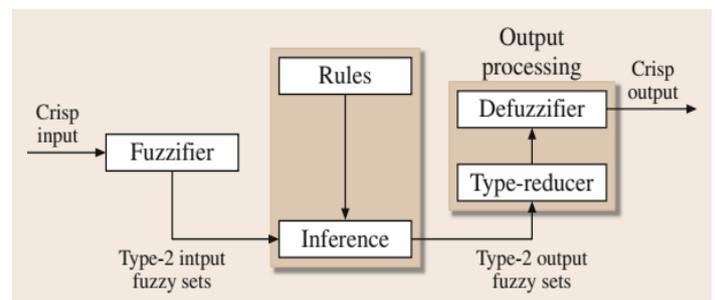


Figure 3: Type-2 FLC block diagram

IV. CONTROLLER DESIGN

The controller is designed by Matlab (R2014a) Simulink using a computer with Intel core i5 @ 253 GHz CPU, 4GB of RAM and 64-bit Windows 7 Operating system as shown in Figure 4.

The first input of FLC is the error and second input is the change of error. As shown in Figure 2.

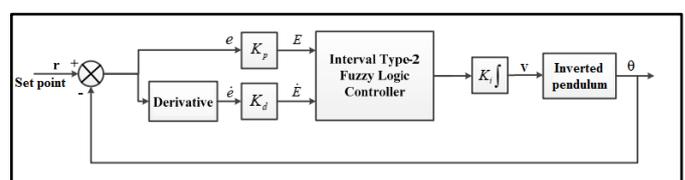


Figure 4: Fuzzy logic controller system

The goal of fuzzy controller in this paper is to control on an inverted pendulum by used a digital pendulum control experiments system 33-936S as shown in Figure 5. If the pendulum tends one of two sides, the controller will change the location of the cart and select suitable direction and speed. System inputs are the pendulum angle (theta) and cart position (X), to determine the cart position and fast.

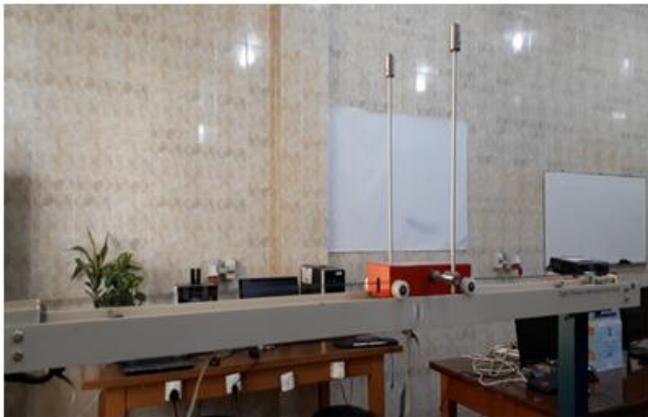


Figure 5: digital pendulum control experiments system 33-936S

The controller has two (multi) input and single output (MISO) with Sugino type-2 fuzzy system. The two-input is the error of pendulum angle and the change in this error and one output is the DC motor voltage (V) are designed in Figure 4.

The membership functions (MF) number of both two inputs are same (7 MF), as shown in Figure 6.

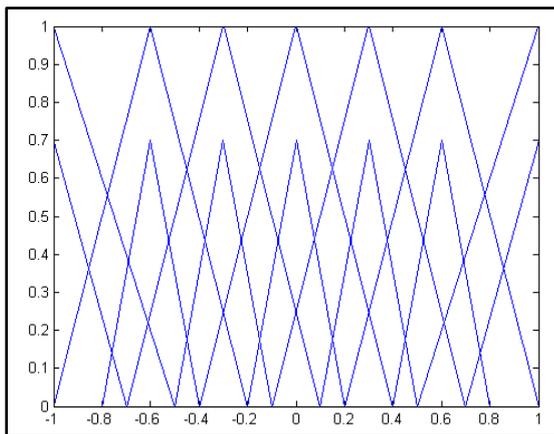


Figure 6: Fuzzy type-2 membership functions for Input variable error and change error

The linguistics description of membership functions are abbreviate as shown in Table 2 for help to keep the linguistic description short but precise and the fuzzy rule base is shown in Table 3.

TABLE 2
Linguistics description abbreviation

Item	linguistics description	Linguistics abbreviation
1	NB	Negative Big
2	NM	Negative Medium
3	NS	Negative Small
4	ZE	Zero Error
5	PS	Positive Small
6	PM	Positive Medium
7	PB	Positive Big

TABLE 3
Fuzzy rule bas

EC E	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NB	NB	NM	ZE
NM	NB	NB	NB	NB	NM	ZE	PM
NS	NB	NB	NB	NM	ZE	PM	PB
ZE	NB	NB	NM	ZE	PM	PB	PB
PS	NB	NM	ZE	PM	PB	PB	PB
PM	NM	ZE	PM	PB	PB	PB	PB
PB	ZE	PM	PB	PB	PB	PB	PB

V. OPTIMIZATION WITH EVOLUTIONARY ALGORITHMS

Selection of the optimal (best) parameters for system controller is a challenging task. Sometimes the results may be bad not because the controller is bad design, but because not good select of the parameters values.

Evolutionary algorithms researchers look up at several natural creatures; they can note that how biological creatures develop and lived in their environment to develop an artificial system acting the same procedure. These branches of science are so called bionic (biological electronic). E.g., the airplane is invented idea of how the birds fly, the radar acts as the bat, submarine comes from fish creating, est. Thus a mechanism of

some optimization algorithms based on nature, e.g., Genetic Algorithm (GA) comes from Charles Darwin’s theory.

This paper presents three types of evolutionary optimization algorithms, first (GA), second (ACO), third (SSO).

5.1 Genetic algorithm (GA)

The principle lines of the genetic algorithm is come from the biological development rules [20]. GA is an efficient evolutionary optimization method with the capability of optimizes complex systems. To carry out a genetic algorithm, the codes of the decision parameters set described primary solution in binary way (0 and 1) or double string or "chromosome". GA differs from non-evolutionary optimization algorithms [21]. GA is probabilistic and not deterministic, additional to it acts with a coding of solution set, instead of the solutions themselves. Also, it searches from a pop of solutions, instead of a single solution. Lastly, GA deals with the cost function without derivatives.

5.2 GA steps

The implementation of GA [20] showed in Figure 7 and as follows:

1. Find the initial pop.
2. Find the fitness function of the pop.
3. Re-production the pop. By use the fittest parents of the final generation.
4. Find the crossover point, by using random way.
5. Find if mutation is occurring and if so on, what is the result of it?
6. Go to step 2 with the new pop. Until the (X) condition be true.

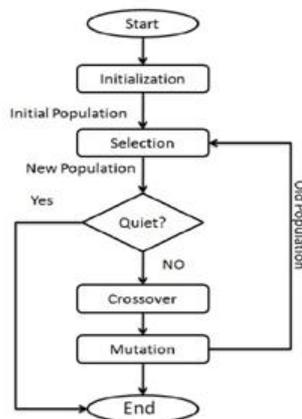


Figure 7: Genetic algorithm (GE) flow chart

5.3 Particles Swarm Optimization (PSO)

(PSO) is an evolutionary stochastic optimization algorithm that based on population guided by intelligent

swarms behavior of some animals such as birds flocks or fish schools [23].

Particle swarm optimization algorithm can be briefly explain as follows: It is searching operation by use a swarm, such that each single element in the swarm is called (a particle) and each particle may be include probable solution of the optimized case in search space, and PSO can keep the best global position of the swarm and that of its particle himself, and memorize the velocity also. In each iteration, the particles data is evaluated for to adjusting the velocity, which is used to calculate the new local position of the particle. Particles positions and velocities is changing constantly in the demanded search space, until they reach to optimal state. There is a unique communication among variant dimensions of the search space is present by the objective functions. The experimental searches showed that the PSO algorithm is a successful optimization tool. Fig. 8 showed the flowchart of the PSO algorithm [23].

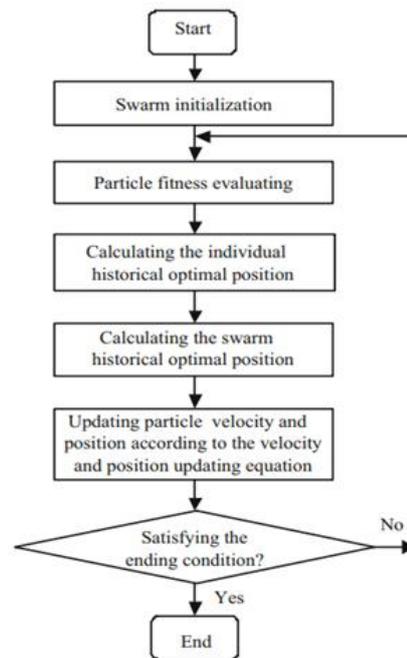


Figure 8: Particle swarm optimization (PSO) Flow Chart

5.4 Ant Colony Optimization (ACO)

The ant colony optimization (ACO) algorithm inspired by the food searching behavior of ants [22]. The scientists studied the complex behavior of ant colony and find that these behavior patterns give us a model for solving complex optimization problems. ACO algorithm is an example for designing evolutionary algorithms for optimization problems. ACO study discrete and continuous optimization problems which is suitable to find approximate solutions. The algorithm that comes from food searching behavior of ant colonies has

been applied to solve complex optimization problems. Now, ACO is very powerful and certificated algorithmic.

Ants communicate by depositing pheromones along paths. Other ants can be smell this pheromone, and its existing effecting the select of their roads, this is meaning, the ants tend to tracing high pheromone concentrations. The pheromone laying down on the paths forms a pheromone roads, which, determined a good resources of food that have been previously determined by other ants. The ACO using the adaptively adjusting the pheromone on paths at all points and as shown in Figure8Select of this points is based on a probability way. The ants are guided by a probability to select their best path, known as a tour.

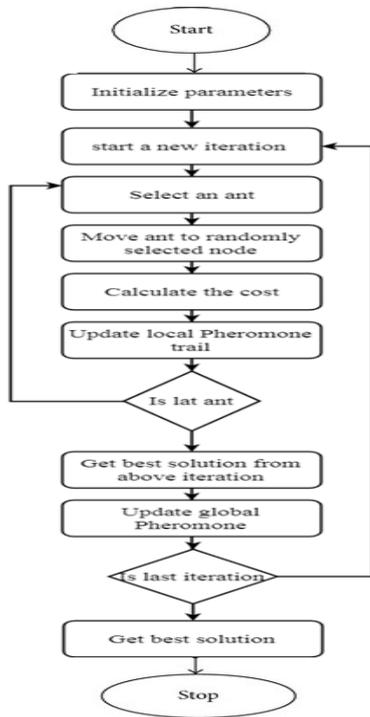


Figure 9: ACO Flow Chart

5.6 Social Spider Optimization (SSO)

SSO is an evolutionary optimization method, its idea inspired from its cooperative properties in the spider colony society [24]. In the SSO, the space of elements is a group of spiders that behave together simulate to the natural social of a spider colony. In majority of evolutionary swarm algorithms, each elements of colony are formed by the similar behavior and characteristics, but, SSO use two distinct elements: female and male. Thus, the gender determine the role, each single element is act as a distinct operation that simulate its nature in the colony spiders. This elements separation improves the significant defects that exist in most evolutionary algorithms. So the SSO has been improved and implemented in several engineering fields.

5.5 Ant colony procedure

The ants walking between the colony and food source, deposit on the paths a substance so called pheromone [22].

The algorithm assumes that the elements act as a common spiders and all supposed solution behave a single spider. SSO designed to deals with a nonlinear problem with some constraint as showed in the following formula:

$$\min.: f(x), \quad x = (x^1, x^2, x^3, \dots, x^d) \in R^d; \quad x \in X$$

Such that:

$$f: R^d \rightarrow R \text{ is a non-linear,}$$

$X = \{x \in R^d | I_h \leq x \leq u_h, h = 1, \dots, d\}$ is a minimize productized search space whose limited with a low (I_h) and by up (u_h) boundaries.

SSO algorithm uses a search space (S) from (N) supposed ways to find optimum solutions for the search problem. To simulate a real colony, every one solution appear as an element location while the total society appear as the problem to space X, the number of female spiders (N_f) select by random way between (65–95)% from a total space S, but the remainder (N_m) is represent a male spiders

$$(N_m = S - N_f).$$

Thus, the set (F_s) represent a female elements($F_s = \{fs_1, fs_2, \dots, fs_{N_f}\}$), thus, the Ms group, represent the male elements, ($Ms = \{ms_1, ms_2, \dots, ms_{N_m}\}$),

Such that

$$S = F_s \cup Ms$$

$$S = \{s_1, s_2, \dots, s_N\}$$

In SSO each one element take weight value (w_{e_i}) with respect to its fitness function, this weight is calculated by:

$$w_{e_i} = \frac{fit_i - Worst}{Best - Worst}$$

Such that:

(fit_i):- is a fitness for ($i - th$) element location, $\{i \in (1, \dots, N)\}$.

Best: - is a best fitness value of entire space.

Worst: - is a worst value of fitness for entire population.

Exchanging information transition is the main technique for the social spider algorithm. This exchange performed by vibrations performs in the spider web, this vibration from spider (i) that a spider (j) perceives it, will emulate and models by:

$$V_{i,j} = W_{e_j} e^{d_{i,j}^2}$$

Such that:

(W_{e_j}):- is a ($j - th$) spider weight.

(d):- is a distance from the first spider (i) to second spider(j).

Each first element (i) understand (3) ways only for web vibration, ($V_{i,j}, V_{i,b}$, and $V_{i,f}$),

Such that:

($V_{i,n}$) :- represent a vibration that performed from most near element n by an upper weighting according to ($W_n - W_i$).

($V_{i,f}$) :- is performed by the most near female element, it's applicable if (i) spider represent male element.

($V_{i,b}$) :- is performed by a best element in the space (S).

With SSO, a population of elements space is started implement from a first stage ($k=0$) to a fixed amount of loops ($k=i$). According to the gender, all spiders are managed by a various set of evolutionary processes. With female element, the new location ($f s_i^{k+1}$) is obtained by updating the current element location ($f s_i^k$). The updating operation is controlled randomly by using a probability factor (Pf) and the moving is performed with other spiders additional to that, its vibrations send with the search space.

$$f(s_i^{k+1}) = \begin{cases} f s_i^k + a \cdot V_{i,n} \cdot (s_n - f s_i^k) - \beta \cdot V_{i,b} \cdot (s_n - f s_i^k) + c \cdot \left(rand - \frac{1}{2} \right) & \text{With } (1 - P_f) \text{ probability} \\ f s_i^k + a \cdot V_{i,n} \cdot (s_n - f s_i^k) + \beta \cdot V_{i,b} \cdot (s_n - f s_i^k) + c \cdot \left(rand - \frac{1}{2} \right) & \text{With probability } (P_f) \end{cases}$$

Such that: a , β , c , and $rand$ are randomly selected values $\in [0,1]$.

(k) is the iteration number.

(s_n) And (s_b) is an individual element symbolize to the nearest element with a weight upper than ($f s_i^k$) and they are the best element in the commune social spider respectively.

And also, the male element is categorized into two types: [dominant (D) and non-dominant (ND)].

The male element that its fitness value are the best with measured to the total male set will integrate with the group. So, the (ND) group is constructed by the remainder of the male elements.

With SSO optimization the (male) elements (ms_i^K) are acted with the following optimum model:

$$(fs_i^{K+1}) = \begin{cases} ms_i^K + a \cdot V_{i,f} \cdot (s_f - ms_i^K) + c \cdot \left(rand - \frac{1}{2} \right) \text{if } ms_i^K \in D \\ ms_i^K + a \cdot \left(\frac{\sum_{h \in ND} ms_i^K \cdot W_h}{\sum_{h \in ND} W_h} - ms_i^K \right) \text{if } ms_i^K \in ND \end{cases}$$

a, c and $rand$:-randomly selected numbers $\in [0,1]$.

s_f :-the nearest female element to the male spider(i).

With the social spider algorithm the (mating operator) is applied between the dominant male spider (m_d) and female element in determined domain(r), to produce a new spider (s_{new}). The weight of every element determines the probability of the influence to every element in(s_{new}).

The spider with the biggest weight has more probability to effect the new spider(s_{new}). When a new element is produced, the rest of the population is compared with it, when the new element is best than worst element in the population, the worst element is exchanged with(s_{new}). Else its neglected. Figure 9 shows the complete evolutionary operation by use flow chart way [24].

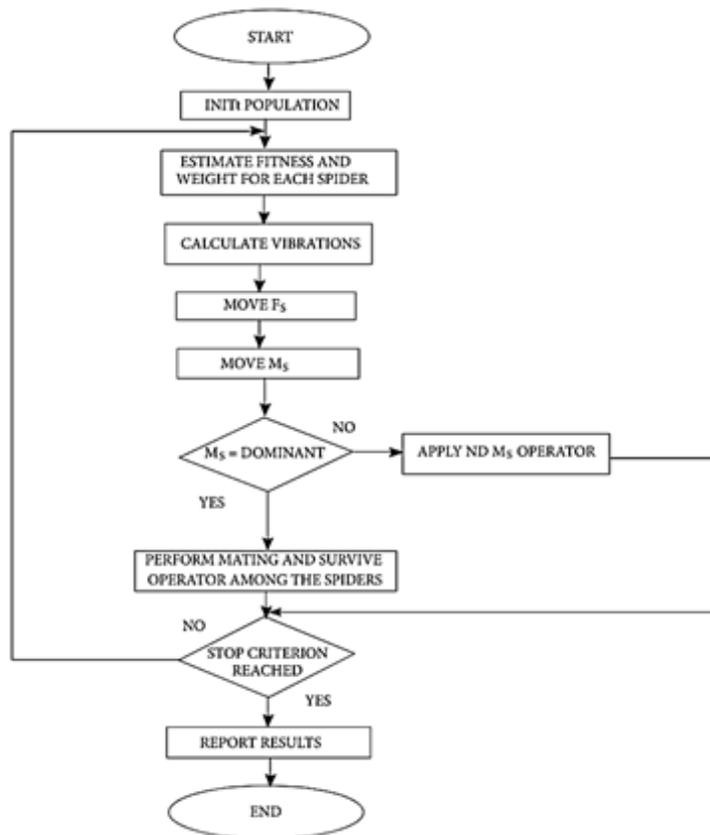


Figure 10: Social Spider Optimization (SSO) flow chart

VI. RESULTS AND DISCUSSIONS

By applying the fuzzy type-2 controller on the mathematical model of inverted pendulum which is obtained in section 2 by using an evolutionary optimization algorithm then the results changed according to the type of optimization algorithm.

The design of the experimental digital pendulum instrument that is used presents two equilibrium points of the pendulum angle, the first one is ($\theta=0$) it means the pendulum is upright (**inverted pendulum**) and the second point is ($\theta=\pi$) it means the pendulum is freely hanging (**crane pendulum**). In this paper, the pendulum angle always starts from ($\theta=\pi=3.14$) that is to say it starts from crane pendulum until it reaches to ($\theta=0$) (Inverted pendulum) under the effect of different controllers.

The first result is applied classical PID controller with the ordinary method of parameters tuning which is (Ziegler-Nichols), the angle of pendulum in this method has a (16 sec.) of oscillation time until it reaches to settling time and big values of overshoot and undershoot as shown in *Figure 11*.

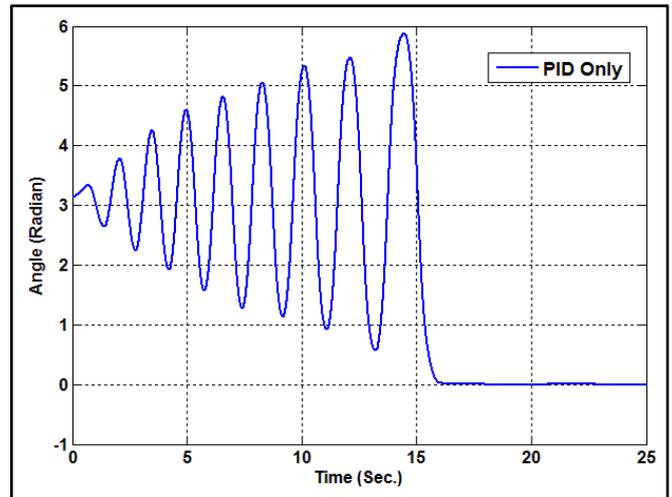
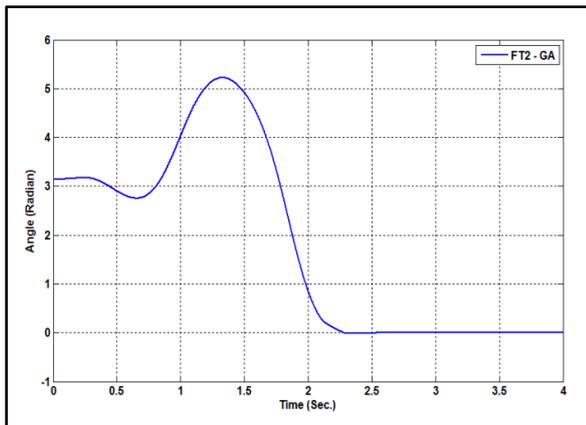
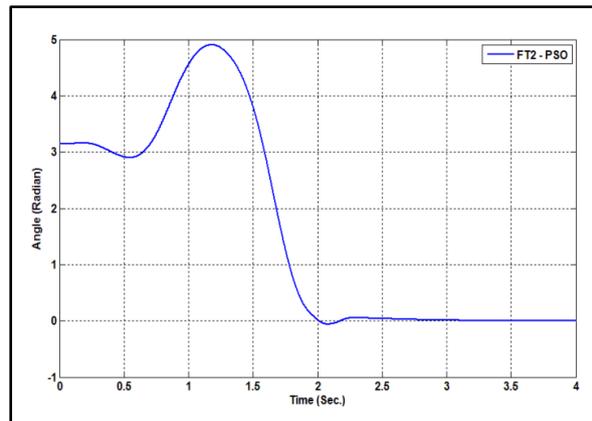


Figure 11: The angle response of PID with Ziegler-Nichols tuning

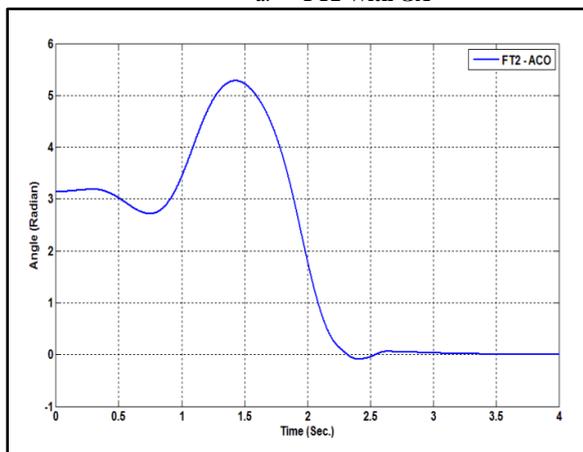
The result of using FT2 controller with tuning the parameters by four evolutionary algorithms Genetic algorithm (GA), Particle swarm optimization (PSO), Ant Colony Optimization (ACO), Social spider optimization (SSO) shows in *Figure 12*.



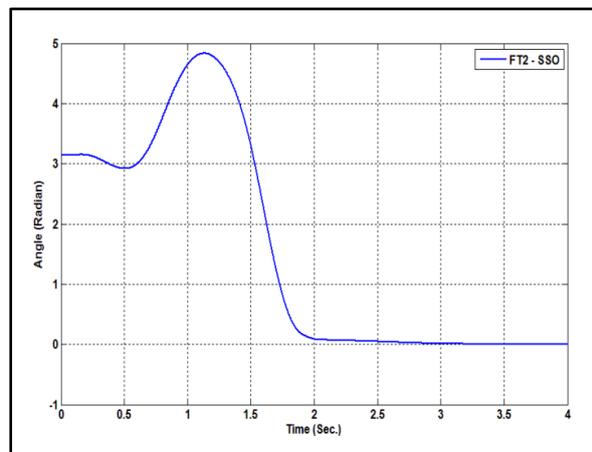
a. FT2 With GA



b. FT2 with PSO



c. FT2 with ACO



d. FT2 with SSO

Figure 12: Comparison FT2 with four evolutionary optimization algorithms

The result in the Figure 13 prove that the using of fuzzy type-2 controller with GA,PSO, ACO, SSO tuning methods (evolutionary optimization algorithms) on the inverted pendulum plant causes distinguished influences in system response, there is a significant change when compared with the fuzzy type-1 controller in the same plant. There is clear decreasing in the settling time and oscillation also, there is reduction in the maximum overshoot and undershoot, but there is increasing in the time of parameter finding (tuning time) as shown in Table 4.

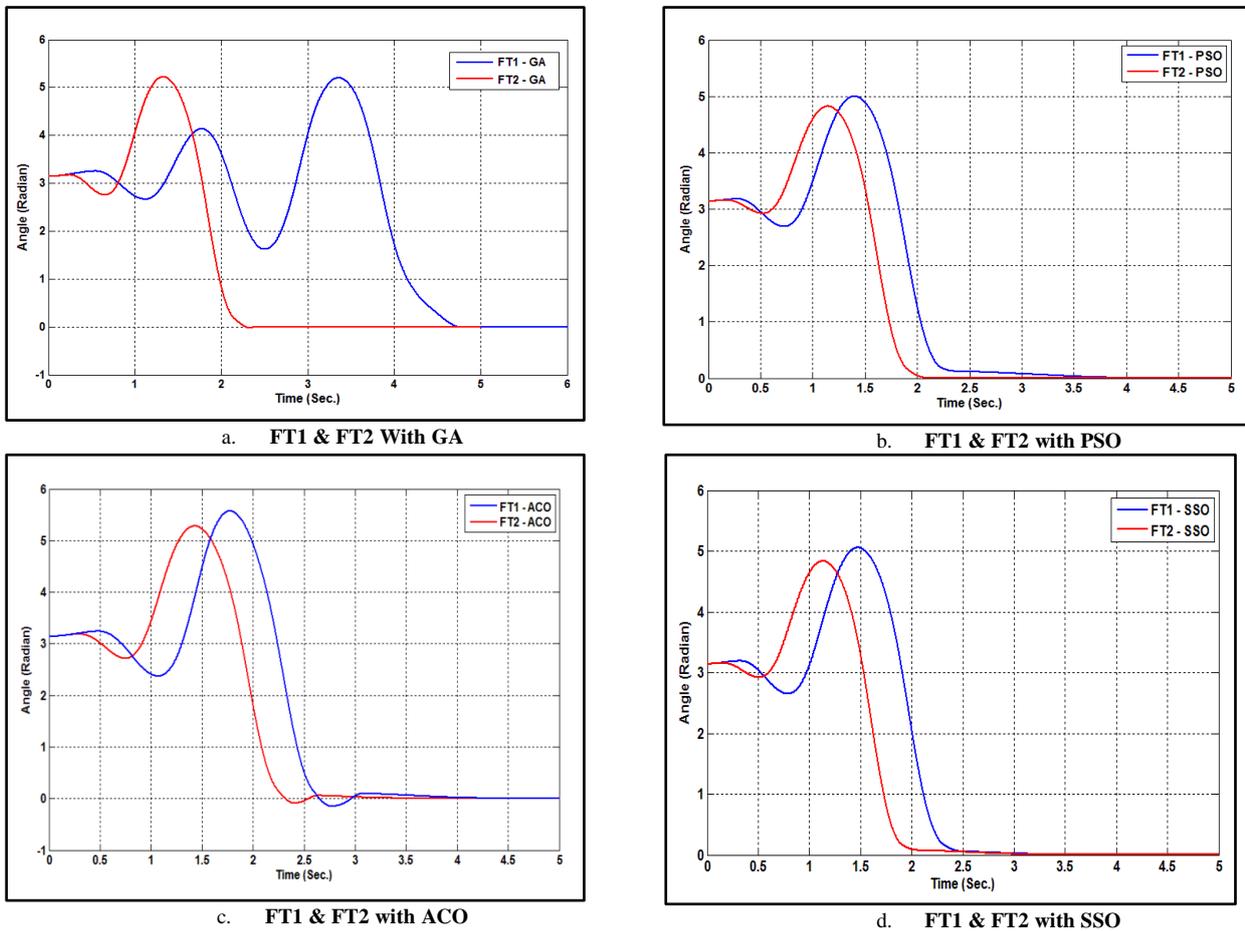


Figure 13: Comparison between the FT1 and FT2withfour evolutionary optimization algorithms

The comparison among the four evolutionary optimization algorithms (GA,PSO, ACO, SSO) with FT-2 controller is shown in Figure 14. The primary result demonstrates the SSO is the best algorithm.

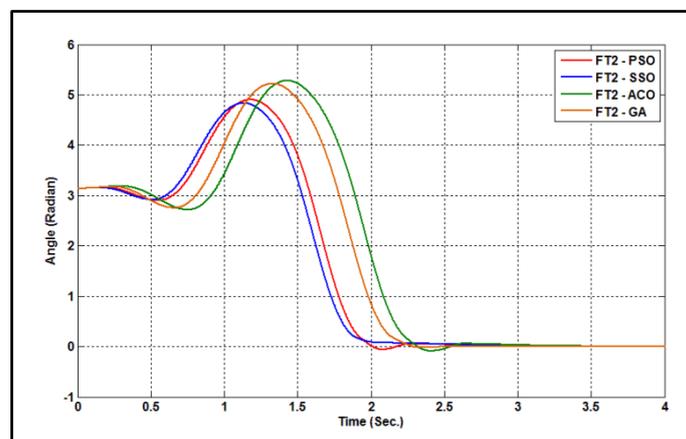


Figure 14: comparison among the four optimization algorithms (GA, PSO, ACO, SSO)

The detailed comparisons of FT-1 & FT-2 controllers with all four algorithms (GA, PSO, ACO, SSO) is shown in Table 4. The comparison includes time response characteristics, fuzzy-PID gains parameters and tuning time for all algorithms. In general, it appears clearly that the time response of the FT-2 controllers is better than the FT-1 for all four algorithms. For the FT-2 controller the SSO achieved the best results in settling time, Peak time and peak value the PSO achieved the best results in less rise time and overshoot. The least optimization time (tuning time) is in (SSO) for both FT-1 & FT-2.

It shows clearly the SSO is the best optimization algorithm in majority of time response characteristics and the tuning time.

TABLE 4
Steady state responses comparison

Title	GA		PSO		ACO		SSO	
	FT1	FT2	FT1	FT2	FT1	FT2	FT1	FT2
Rise Time (sec.)	3.5508	1.5428	1.5879	0.2801	1.7483	1.5706	1.6202	0.2925
Settling Time(sec.)	4.5164	2.1127	2.1896	1.9047	2.5489	2.2040	2.2652	1.8656
Overshoot (%)	1.9900	1.9889	1.9999	1.9687	2.4026	2.0969	1.9955	1.9990
Peak Time (sec.)	3.3605	1.3305	1.4000	1.1805	1.7755	1.4275	1.4755	1.1335
Peak value	5.2015	5.2231	5.0011	4.9032	5.5737	5.2767	5.0556	4.8342
Kp	28.758	39.981	30.000	20.530	2.798	9.3286	17.000	34.4673
Ki	29.9095	29.232	2.912	30.802	4.84527	30.0495	2.8986	30.2667
Kd	38.8786	38.148	0.594	20.000	5.6986	6.74899	0.5870	37.3689
Tuning time (min.)	1827	2417.999	199.795	469.950	210.5	576.267	70.287	276.022

VII. CONCLUSIONS

The fuzzy type-1 and type-2 controllers has been used to control the inverted pendulum on a cart by using a digital(pendulum control experiments system 33-936S) and the parameters of controller (Gains) is tuned with four evolutionary optimization(GA), (PSO), (ACO) and (SSO).

The result of tuned FT-1/FT-2 with evolutionary optimization is compared with all four evolutionary optimization. The comparisons carried out between the controllers of fuzzy type-1and fuzzy type-2 as well as between several optimization algorithm with the fuzzy typ-2 itself. The result appears that the time response characteristics of the FT-2 controllers are better than the FT-1 for all four algorithms. And the FT-2 controller with SSO achieved the best results in settling time. The least tuning time is in (SSO) for both FT-1 & FT-2 controllers. It's clearly the SSO is the best optimization algorithm.

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