

# Application of Three Parameter Model to Concave and Convex Curves

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**Abstract** – Many linear and non-linear models have been proposed by various authors for the study of approximation of non linear models by statistical models which are linear in its parameters. Several non linear models have been suggested for the data sets which are prima facie appear like convex or concave curves. Smith (1938), jessen (1942) and Hansen et.al. (1953), Misra (2010), Shukla et.al. 2018), Tiwari (2022) have proposed various non-linear models for the comparative study for such data sets and they compared the parameters like coefficient of determination and sum of squared error. In this paper I have suggested an application of a three parameter linear model to compare these parameters of interest for the data sets which form convex or concave curves.

**Keywords:** Convex Curve, Concave Curve, Linear model, non-linear model, three parameter model, coefficient of determination, sum of squared error.

## I. INTRODUCTION

The intrinsically nonlinear model of the form,

$$Y = \theta_1(1 - e^{-\theta_2 t}) + \epsilon \quad (1)$$

is fitted to various data sets reported in Draper and Smith (1981).  $\theta_1$  and  $\theta_2$  are parameters of the model (1) and  $t$  is treated as independent variable.

$\epsilon$ 's are i.i.d.r.v. distributed normally with mean zero and fixed variance  $\sigma^2$ . The deterministic part of model (1) can be written as

$$Y = \theta_1(1 - e^{-\theta_2 t}) \quad (2)$$

The relation (2) belongs to the family of concave/convex curves and Ratkowsky (1989) has not categorized the relation (2) as close-to-linear model.

The form (2) can be written as

$$Y = \theta_1(1 - \beta^t) \quad (3)$$

where,

$$\theta_2 = -\log(\beta) \quad (4)$$

Relation (3) can be categorized as parameterization of form (2). The nonlinearly appearing parameter in (3) is  $\beta$  and due to the term  $\beta^t$ , the direct use of least squares method is not possible. As discussed earlier, the term  $\beta^t$  is approximated by an inverse term, resulting into model of the form,

$$Y = a + bt + \frac{c}{t}, t > 0 \quad (5)$$

Approximating the relation (3) and hence relation (2) which is parameterization of (3), on the lines of suggestions made earlier.

Table given below, reports the fitted forms for the model (5) on various data sets in Draper and Smith (1981). The data sets are listed as under:

## II. RESULT TABLE

Fitted curves,  $R^2$  and  $s^2$  values for the functional forms (5)

Source	Functional Forms					
			(5)			(2)
	a	b	c	$R^2$	$S^2$	$S^2$
<b>Draper &amp; Smith (1981)</b>						
Ex. L, Pg. 522-524						
Data Set 1	130.7126	11.6284	-62.7046	0.9816	50.8199	252.0
Data Set 2	1.1148	0.1157	-0.8289	0.9689	0.0173	0.0262
Data Set 3	421.6973	51.9357	-316.1272	0.9773	1854.73	3376.5
Data Set 4	11.8338	1.8019	-5.6243	0.8601	8.1535	17.004
Data Set 5	9.9117	0.5962	-6.1770	0.9922	0.1285	0.866
Data Set 6	14.9176	0.8368	-8.7852	0.9891	0.3601	3.716
Data Set 7	158.054	7.7589	-57.3651	0.9656	112.04	1168.0
Data Set 8	16.1349	0.6220	-8.8585	0.9501	8.94	25.990
Data Set 9	6828.96	875.30	-2994.63	0.99	18.66	68349.0

## III. CONCLUSION

It is seen from the table that relation (5) fits the data well and  $s^2$  values for the form (5) are much lower than  $s^2$  values for the form (2) in all cases.

## REFERENCES

- [1] Aggrey S E (2009) on Logistic nonlinear mixed effect model for estimating growth parameters. Journal of Poultry Science (2009) vol. 88, pg. 276-280.
- [2] Akbar Atif, Pusha G.R., Aslam, Muhammad Warsi S.K.A. (2009). "Statistical Modeling for citrus yield in Pakistan". European journal of Scientific Research ISSN 1450-216X vol. 31 No. 1(2009), pp. 52-58.
- [3] Cochran W G (1977). Sampling Techniques. John Wiley & Sons.
- [4] Jessen R J (1978). Statistical Survey Techniques. Wiley, New York.
- [5] Mahir A, Rozita W D W, Khairiah J and Ismail B S (2009). Fitting a Nonlinear Regression Model to Gauge Heavy Metal Uptake in Spinach (*Amaranthushybridus L.*). American-Eurasian J. Agric. & Environ. Sci., 5(2): 236-243, 2009.
- [6] Misra G C (1992). "Some Estimation Precedures in Non-Linear Statistical Models". The Unpublished Thesis of Ph. D., Kanpur University, Kanpur.
- [7] Moh. Saat N Z, Jemain A A, Al-Mashoor S H A (2008). A Comparison of Weibull and Gamma Distribution in Application of Sleep Apnea. Asian Journal of Mathematics and Statistics 1 (3): 132-138, 2008.
- [8] Mukhopadhyay, P. (1998). Theory and Methods of Survey Sampling. Prentice Hall of India Pvt. Ltd. New Delhi, India.
- [9] Nygaard R W, Katharina V E and Ronald A S (2008). Models of reading performance in older adults with normal age-related vision. Journal of rehabilitation Research & Development, Volume 45, Number 6, 2008, Pages 901-910.
- [10] Singh N O, Alam M W, Paul A K and Kumar S (2009). Length-Weight Relationship and Growth Pattern of Tor Putitora (Hamilton) under Monoculture and Polyculture Systems: A Case Study. J. Ind. Soc. Statist. 63(1), 2009: 85-89.
- [11] Shukla et.al. (2020). New Linear Model for Optimal Cluster Size in Cluster Sampling. Statistics in Transition new Series 21(2): page 189-200.
- [12] Tiwari R.B. (2022), Application of Three Parameter Model For Determination of optimum Cluster Size, International Journal of Research and Analytical Reviews ijar, Vol 09, Issue 03, July, 2022. Page: 134-143.



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