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Fuzzy Quotient-3 Cordial Labeling on Some Cycle Related Graphs

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Abstract **- Let G be a simple, finite, non-trivial graph with p vertices and q edges. V and E be the vertex set and edge set of G respectively. Let the function $\sigma: V \to [0,1]$ defined by $\sigma(v) = \frac{\gamma}{10}, \gamma \in Z_4 - \{0\}$ and for each$\omega\gamma \in E$, the induced function $\mu \to [0,1]$ defined by $\mu(\omega\gamma) = \frac{1}{10}\left\lceil\frac{3\sigma(\omega)}{\sigma(\gamma)}\right\rceil$ where $\sigma(\omega) \leq \sigma(\gamma)$ such that $|v_\sigma(i) - v_\sigma(j)| \leq 1$ and $|e_\mu(i) - e_\mu(j)| \leq 1$, where $v_\sigma(k)$represents the number of vertices getting the label $k \in \left\{\frac{\gamma}{10}, \gamma \in Z_4 - \{0\}\right\}$and $e_\mu(k)$represents the number of edges getting the label $k \in \left\{\frac{\gamma}{10}, \gamma \in Z_4 - \{0\}\right\}$. Then the function $\sigma$is called Fuzzy quotient-3 cordial labelling and the graph $G(V, E)$ is a fuzzy quotient-3 cordial graph. Fuzzy quotient -3 cordial labeling on some cycle - related graphs are investigated and the works are presented in this paper.**

Keywords: Vertex duplication, Edge duplication, Mutual duplication, Vertex switching, joint sum, Fuzzy quotient-3 cordial graph.

I. INTRODUCTION

Based on certain conditions, allotting values to vertices or edges or both vertices and edges of the graph are called labeling of a graph. This technique was introduced by Rosa (1967) or Graham & Sloane (1980). Labeling the graph creates a lot of interest and motivation among many researchers. Joseph A. Gallian gives a complete survey on the graph labeling Motivated by these labelings, we introduced a fuzzy quotient-3 cordial labeling and investigate some graph families, and proved to be a fuzzy quotient-3 cordial.

In this paper the Fuzzy quotient-3 cordial labeling on some path –related graphs are investigated and proved that the graphs are fuzzy quotient-3 cordial.

II. DEFINITIONS

2.1 Duplication of a vertex

Duplication of a vertex v_k of a graph G produces a new graph $D_v(G)$ by adding a new vertex v'_k in such a way that $N(v_k) = N(v'_k)$[11].

2.2 Duplication of an edge

Duplication of an edge $v_i v_{i+1}$ of a graph G produces a new graph $D_e(G)$ by adding a new edge $v'_i v'_{i+1}$ in such a way that $N(v'_i) = N(v_i) \cup \{v'_{i+1}\} - \{v_{i+1}\}$ and $N(v'_{i+1}) = N(v_{i+1}) \cup \{v'_i\} - \{v_i\}$[12].

2.3 Duplication of a vertex by an edge

Duplication of a vertex v_k by a new edge $e = v'_k v''_k$ in a graph G produces a new graph G' such that $N(v'_k) \cap N(v''_k) = v_k$[11].

2.4 Duplication of an edge by a vertex

Duplication of an edge $e = uv$ by a new vertex w in a graph G produces a new graph G' such that $N(w) = \{u, v\}$[12].

2.5 Mutual duplication of a pair of vertices

Consider two copies of cycle C_n. Then the mutual duplication of a pair of vertices v_k and v'_k respectively from each copy of cycle C_n produces a new graph G such that $N(v_k) = N(v'_k)$[13].

2.6 Mutual duplication of a pair of edges

Consider two copies of cycle C_n and let $e_k = v_k v_{k+1}$ be an edge in the first copy of C_n with $e_{k-1} = v_{k-1} v_k$ and $e_{k+1} = v_{k+1} v_{k+2}$ be its incident edges. Similarly let $e'_k = u_k u_{k+1}$ be an be an edge in the second copy of C_n with $e'_{k-1} = u_{k-1} u_k$ and $e'_{k+1} = u_{k+1} u_{k+2}$ be its incident edges. The mutual duplication of a pair of edges e_k, e'_k respectively from two copies of cycle C_n produces a new graph G in such a

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way that $N(v_k) \cap N(u_k) = \{v_{k-1}, u_{k-1}\}$ and $N(v_{k+1}) \cap N(u_{k+1}) = \{v_{k+2}, u_{k+2}\}$[13].

2.7 Vertex switching

A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing all edges incident to v and adding edges joining v and adding edges joining v to every vetex not adjacent to v in G[11].

2.8 Joint sum

Consider two copies of a graph G and define a new graph known as joint sum is the graph obtained by connecting a vertex of first copy with a vertex of second copy by an edge [11].

2.9 Fuzzy quotient-3 cordial graph

A simple, finite, non-trivial graph G with p vertices and q edges. V and E be the vertex set and edge set of G respectively. Let the function $\sigma: V \to [0,1]$ defined by $\sigma(v) = \frac{\gamma}{10}, \gamma \in Z_4 - \{0\}$and for each $\omega\gamma \in E$, the induced function $\mu \to [0,1]$defined by $\mu(\omega\gamma) = \frac{1}{10}\left\lceil \frac{3\sigma(\omega)}{\sigma(\gamma)} \right\rceil$ where $\sigma(\omega) \leq \sigma(\gamma)$ such that $|v_\sigma(i) - v_\sigma(j)| \leq 1$ and $|e_\mu(i) - e_\mu(j)| \leq 1$, where $v_\sigma(k)$ represents the number of vertices getting the label $k \in \left\{\frac{\gamma}{10}, \gamma \in Z_4 - \{0\}\right\}$and $e_\mu(k)$represents the number of edges getting the label $k \in \left\{\frac{\gamma}{10}, \gamma \in Z_4 - \{0\}\right\}$. Then the function $\sigma$is called Fuzzy quotient -3 cordial labelling and the graph $G(V, E)$ is a fuzzy quotient-3 cordial graph [14].

III. RESULTS

Theorem 3.1 A graph $D_v(C_\eta)$ admit fuzzy quotient-3 cordial labeling.

Proof:

Let $D_v(C_\eta)$ be the graph obtained by duplicating an arbitrary vertex of C_η. Let this arbitrary vertex be x_1 and the newly added vertex be x_1'.

$V(D_v(C_\eta)) = \{x_\iota : 1 \leq \iota \leq \eta\} \cup \{x_1'\}$ and $E(D_v(C_\eta)) = \{x_\iota x_{\iota+1} : 1 \leq \iota \leq \eta - 1\} \cup \{x_1 x_\eta\} \cup \{x_1' x_2\} \cup \{x_1' x_n\}. p = \eta + 1$ and $q = \eta + 2$.

To define $\sigma: V(D_v(C_n)) \to [0,1]$ the following cases are to be considered.

Case 1:$\eta \equiv 0(mod\ 6)$
$\sigma(x_1') = 0.1$
$\sigma(x_\iota) = 0.1 \quad \iota \equiv 1,2(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(x_\iota) = 0.2 \quad \iota \equiv 4,5(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(x_\iota) = 0.3 \quad \iota \equiv 0,3(mod\ 6) \quad 1 \leq \iota \leq \eta$

Case 2:$\eta \equiv 1(mod\ 6)$
$\sigma(x_1') = 0.1$
$\sigma(x_\iota) = 0.1 \quad \iota \equiv 4,5(mod\ 6) \quad 1 \leq \iota \leq \eta - 1$
$\sigma(x_\iota) = 0.2 \quad \iota \equiv 1,2(mod\ 6) \quad 1 \leq \iota \leq \eta - 1$
$\sigma(x_\iota) = 0.3 \quad \iota \equiv 0,3(mod\ 6) \quad 1 \leq \iota \leq \eta - 1$
$\sigma(x_\eta) = 0.3$

Case 3:$\eta \equiv 2(mod\ 6)$
$\sigma(x_1') = 0.2$ and the labeling of x_i for $1 \leq \iota \leq \eta - 2$is same as in Case 2 and $\sigma(x_{\eta-1}) = 0.3$, $\sigma(x_\eta) = 0.1$

Case 4:$\eta \equiv 3(mod\ 6)$
$\sigma(x_1') = 0.2$ and the labeling of x_i for $1 \leq \iota \leq \eta$is same as in Case 1

Case 5:$\eta \equiv 4(mod\ 6)$
$\sigma(x_1') = 0.2$ and the labeling of x_i for $1 \leq \iota \leq \eta - 1$is same as in Case 1 and $\sigma(x_\eta) = 0.3$

Case 6:$\eta \equiv 5(mod\ 6)$
$\sigma(x_1') = 0.3$ and the labeling of x_i for $1 \leq \iota \leq \eta$is same as in Case 1

The following table gives the number of vertices and edges which are labelled with $k \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$

Table 1: -$v_\mu(i)$ for the graph $D_v(C_n)$

***Nature of* η**	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$
$\eta \equiv 0(mod\ 6)$	$\frac{p-1}{3}+1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$
$\eta \equiv 1,4(mod\ 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3}-1$	$\frac{p+1}{3}$
$\eta \equiv 2(mod\ 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$
$\eta \equiv 3(mod\ 6)$	$\frac{p-1}{3}+1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$
$\eta \equiv 5(mod\ 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$

Table 2: -$e_\sigma(i)$ for the graph $D_v(C_n)$

***Nature of* η**	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$\eta \equiv 0(mod\ 6)$	$\frac{q+1}{3}$	$\frac{q+1}{3}-1$	$\frac{q+1}{3}$
$\eta \equiv 1,4(mod\ 6)$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$\eta \equiv 2(mod\ 6)$	$\frac{q-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3}+1$
$\eta \equiv 3(mod\ 6)$	$\frac{q+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3}-1$
$\eta \equiv 5(mod\ 6)$	$\frac{q-1}{3}$	$\frac{q-1}{3}+1$	$\frac{q-1}{3}$

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From the above table 1 and 2, we can easily find that the conditions of fuzzy quotient - 3 cordial labeling, i.e., $|v_\sigma(i) - v_\sigma(j)| \leq 1$ and $|e_\mu(i) - e_\mu(j)| \leq 1$ for $i \neq j$, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ is satisfied.Hence the duplication of an arbitrary vertex of a cycle $C_n$admits fuzzy quotient - 3 cordial labeling.

Theorem 3.2 The graph $D_e(C_n)$ admits fuzzy quotient-3 cordial labeling.

Proof: Let $D_e(C_n)$ be the graph obtained by duplicating an arbitrary edge of C_n. Without loss of generality let this edge be x_1x_2 and the newly added edge be $x_1'x_2'$.

$V(D_e(C_n)) = \{x_\iota : 1 \leq \iota \leq \eta\} \cup \{x_1', x_2'\}$ and $E(D_e(C_n)) = \{x_\iota x_{\iota+1} : 1 \leq \iota \leq \eta - 1\} \cup \{x_1 x_\eta\} \cup \{x_1' x_\eta\} \cup \{x_2' x_3\}. p = \eta + 2$ and $q = \eta + 3$.

To define $\sigma: V(D_e(C_n)) \rightarrow [0,1]$ the following cases are to be considered.

Case 1: $\eta \equiv 0(mod\ 6)$
$\sigma(x_1') = 0.1, \sigma(x_2') = 0.3$
$\sigma(x_\iota) = 0.1 \quad \iota \equiv 0,1(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(x_\iota) = 0.2 \quad \iota \equiv 3,4(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(x_\iota) = 0.3 \quad \iota \equiv 2,5(mod\ 6) \quad 1 \leq \iota \leq \eta$

Case 2:$\eta \equiv 1,2(mod\ 6)$
$\sigma(x_1') = 0.3$, $\sigma(x_2') = 0.2$ and the labeling of x_ι for $1 \leq \iota \leq \eta$is same as in Case 1.

Case 3: $\eta \equiv 3(mod\ 6)$
$\sigma(x_1') = 0.2, \sigma(x_2') = 0.2$
$\sigma(x_\iota) = 0.1 \quad \iota \equiv 1,2(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(x_\iota) = 0.2 \quad \iota \equiv 4,5(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(x_\iota) = 0.3 \quad \iota \equiv 0,3(mod\ 6) \quad 1 \leq \iota \leq \eta$

Case 4: $\eta \equiv 4(mod\ 6)$
$\sigma(x_1') = 0.2$, $\sigma(x_2') = 0.2$ and the labeling of x_i for $1 \leq \iota \leq \eta - 1$is same as in Case 3 and $\sigma(x_\eta) = 0.3$

Case 5: $\eta \equiv 5(mod\ 6)$
$\sigma(x_1') = 0.3$, $\sigma(x_2') = 0.1$ and the labeling of x_i for $1 \leq \iota \leq \eta$is same as in Case 1.

The following table gives the number of vertices and edges which are labelled with $k \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$.

Table 3: -$v_\sigma(i)$ for the graph $D_e(C_n)$

Nature of η	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$
$\eta \equiv 0(mod\ 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$	$\frac{p+1}{3}$
$\eta \equiv 1,4(mod\ 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$
$\eta \equiv 2(mod\ 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$
$\eta \equiv 3(mod\ 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$
$\eta \equiv 5(mod\ 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$

Table 4: -$e_\mu(i)$ for the graph $D_e(C_n)$

Nature of η	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$\eta \equiv 0(mod\ 6)$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$\eta \equiv 1,4(mod\ 6)$	$\frac{q-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$
$\eta \equiv 2(mod\ 6)$	$\frac{q+1}{3}$	$\frac{q+1}{3} - 1$	$\frac{q+1}{3}$
$\eta \equiv 3(mod\ 6)$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$\eta \equiv 5(mod\ 6)$	$\frac{q+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3} - 1$

From the above table 3 , we can easily find that the conditions of fuzzy quotient - 3 cordial labeling, i.e., $|v_\sigma(i) - v_\sigma(j)| \leq 1$ and $|e_\mu(i) - e_\mu(j)| \leq 1$ for $i \neq j$, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ is satisfied. Hence the graph $D_e(C_n)$admits fuzzy quotient - 3 cordial labeling.

Theorem 3.3 The graph obtained by duplication of an arbitrary vertex by a new edge in cycle C_η admits fuzzy quotient - 3 cordial labeling, except for $\eta \equiv 0(mod\ 6)$.

Proof: Let G be the graph obtained by duplicating an arbitrary vertex of C_n by a new edge. Without loss of generality let this vertex be x_1 and the edge be $x'_1x''_1$. $V(G) = \{x_i : 1 \leq \iota \leq \eta\} \cup \{x'_1x''_1\}$ and $E(G) = \{x_\iota x_{\iota+1} : 1 \leq \iota \leq \eta - 1\} \cup \{x_1 x_\eta\} \cup \{x'_1x''_1, x'_1x_1, x''_1x_1\}$.
$p = \eta + 2$ and$q = \eta + 3$.
To define $\sigma: V(G) \rightarrow [0,1]$ the following cases are to be considered.

Case 1:$\eta \equiv 1,2(mod\ 6)$
$\sigma(x'_1) = 0.1, \sigma(x''_1) = 0.3$
$\sigma(x_\iota) = 0.1 \quad \iota \equiv 4,5(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(x_\iota) = 0.2 \quad \iota \equiv 1,2(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(x_\iota) = 0.3 \quad \iota \equiv 0,3(mod\ 6) \quad 1 \leq \iota \leq \eta$

Case 2:$\eta \equiv 3(mod\ 6)$
$\sigma(x'_1) = 0.1, \sigma(x''_1) = 0.1\ \sigma(x_1) = 0.3$
$\sigma(x_\iota) = 0.1 \quad \iota \equiv 0,1(mod\ 6) \quad 2 \leq \iota \leq \eta$
$\sigma(x_\iota) = 0.2 \quad \iota \equiv 3,4(mod\ 6) \quad 2 \leq \iota \leq \eta$
$\sigma(x_\iota) = 0.3 \quad \iota \equiv 2,5(mod\ 6) \quad 2 \leq \iota \leq \eta$

Case 3:$\eta \equiv 4(mod\ 6)$
$\sigma(x'_1) = 0.1$, $\sigma(x''_1) = 0.3$, $\sigma(x_1) = 0.1$ and the labeling of x_i for $2 \leq \iota \leq \eta$is same as in Case 2.

Case 4:$\eta \equiv 4(mod\ 6)$

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$\sigma(x'_1) = 0.1,$ $\sigma(x''_1) = 0.3$

The labeling of x_i for $1 \le \iota \le \eta - 1$is same as in Case 2 and $\sigma(x_\eta) = 0.2$

The following table gives the number of vertices and edges which are labelled with $k \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$

Table 5: -$v_\sigma(i)$ for the graph-Duplication of a vertex by an edge

Nature of η	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$
$\eta \equiv 1(mod\ 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$
$\eta \equiv 2(mod\ 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3}+1$	$\frac{p-1}{3}$
$\eta \equiv 3(mod\ 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3}-1$	$\frac{p+1}{3}$
$\eta \equiv 4(mod\ 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$
$\eta \equiv 5(mod\ 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3}+1$	$\frac{p-1}{3}$

Table 6: -$e_\mu(i)$ for the graph-Duplication of a vertex by an edge

Nature of η	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$\eta \equiv 1(mod\ 6)$	$\frac{q-1}{3}$	$\frac{q-1}{3}+1$	$\frac{q-1}{3}$
$\eta \equiv 2(mod\ 6)$	$\frac{q+1}{3}-1$	$\frac{q+1}{3}$	$\frac{q+1}{3}$
$\eta \equiv 3(mod\ 6)$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$\eta \equiv 4(mod\ 6)$	$\frac{q-1}{3}+1$	$\frac{q-1}{3}$	$\frac{q-1}{3}$
$\eta \equiv 5(mod\ 6)$	$\frac{q+1}{3}$	$\frac{q+1}{3}-1$	$\frac{q+1}{3}$

From the above table 5 and 6 , we can easily find that the conditions of fuzzy quotient - 3 cordial labeling, i.e., $|v_\sigma(i) - v_\sigma(j)| \le 1$ and $|e_\mu(i) - e_\mu(j)| \le 1$ for $i \ne j$, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ is satisfied. Hence the graph obtained by duplication of an arbitrary vertex by a new edge in cycle $C_n$admits fuzzy quotient - 3 cordial labeling.

Theorem 3.4 The graph obtained by duplication of an arbitrary edge by a new vertex in cycle C_η admits fuzzy quotient - 3 cordial labeling.

Proof:

Let G be the graph obtained by duplicating an arbitrary edge of C_n by a new vertex. Without loss of generality let this edge be x_1x_2 and the vertex be x'_1. $V(G) = \{x_\iota : 1 \le \iota \le \eta\} \cup \{x'_1\}$ and

$E(G) = \{x_\iota x_{\iota+1} : 1 \le \iota \le \eta - 1\} \cup \{x_1 x_\eta\} \cup \{x'_1 x_1, x'_1 x_2\}$.

$p = \eta + 1$ and $q = \eta + 2$.

To define $\sigma: V(G) \to [0,1]$ the following cases are to be considered.

Case 1:$\eta \equiv 0,3,4,5(mod\ 6)$

$\sigma(x'_1) = 0.1$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 0,1(mod\ 6) \quad 1 \le \iota \le \eta$

$\sigma(x_\iota) = 0.2 \quad \iota \equiv 3,4(mod\ 6) \quad 1 \le \iota \le \eta$

$\sigma(x_\iota) = 0.3 \quad \iota \equiv 2,5(mod\ 6) \quad 1 \le \iota \le \eta$

Case 2:$\eta \equiv 1(mod\ 6)$

$\sigma(x'_1) = 0.1$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 0,5(mod\ 6) \quad 1 \le \iota \le \eta$

$\sigma(x_\iota) = 0.2 \quad \iota \equiv 2,3(mod\ 6) \quad 1 \le \iota \le \eta$

$\sigma(x_\iota) = 0.3 \quad \iota \equiv 1,4(mod\ 6) \quad 1 \le \iota \le \eta$

Case 3:$\eta \equiv 2(mod\ 6)$

$\sigma(x'_1) = 0.1$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 2,3(mod\ 6) \quad 1 \le \iota \le \eta - 1$

$\sigma(x_\iota) = 0.2 \quad \iota \equiv 0,5(mod\ 6) \quad 1 \le \iota \le \eta - 1$

$\sigma(x_\iota) = 0.3 \quad \iota \equiv 1,4(mod\ 6) \quad 1 \le \iota \le \eta - 1$

$\sigma(x_\eta) = 0.2$

The following table gives the number of vertices and edges which are labelled with $k \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$

Table 7: -$v_\sigma(i)$ for the graph-Duplication of a edge by an vertex

Nature of η	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$
$\eta \equiv 0(mod\ 6)$	$\frac{p-1}{3}+1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$
$\eta \equiv 1(mod\ 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3}-1$	$\frac{p+1}{3}$
$\eta \equiv 2(mod\ 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$
$\eta \equiv 3(mod6)$	$\frac{p-1}{3}+1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$
$\eta \equiv 4(mod\ 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3}$	$\frac{p+1}{3}-1$
$\eta \equiv 5(mod\ 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$

Table 8: -$e_\mu(i)$ for the graph-Duplication of a edge by an vertex

Nature of η	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$\eta \equiv 0(mod\ 6)$	$\frac{q+1}{3}$	$\frac{q+1}{3}-1$	$\frac{q+1}{3}$
$\eta \equiv 1(mod\ 6)$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$\eta \equiv 2(mod\ 6)$	$\frac{q-1}{3}$	$\frac{q-1}{3}+1$	$\frac{q-1}{3}$
$\eta \equiv 3(mod6)$	$\frac{q+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3}-1$
$\eta \equiv 4(mod\ 6)$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$\eta \equiv 5(mod\ 6)$	$\frac{q-1}{3}+1$	$\frac{q-1}{3}$	$\frac{q-1}{3}$

From the above table 7 and8, we can easily find that the conditions of fuzzy quotient - 3 cordial labeling, i.e., $|v_\sigma(i) - v_\sigma(j)| \le 1$ and $|e_\mu(i) - e_\mu(j)| \le 1$ for $i \ne j$, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ is satisfied. Hence the graph obtained by

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duplication of an arbitrary edge by a new vertex in cycle C_n admits fuzzy quotient - 3 cordial labeling.

Theorem 3.5 The graph obtained by the mutual duplication of a pair of vertices in two copies of cycle C_η admits fuzzy quotient - 3 cordial labeling.

Proof:

Let G be the graph obtained by the mutual duplication of a pair of vertices each respectively from each copy of cycle C_η . Without loss of generality assume that the vertex x_1 from the first copy of cycle C_η and the vertex y_1 from the second copy of cycle C_η are mutually duplicated.

$V(G) = \{x_\iota : 1 \leq \iota \leq \eta\} \cup \{y_\iota : 1 \leq \iota \leq \eta\}$ and $E(G) = \{x_\iota x_{\iota+1} : 1 \leq \iota \leq \eta - 1\} \cup \{x_1 x_\eta\} \cup \{y_\iota y_{\iota+1} : 1 \leq \iota \leq \eta - 1\} \cup \{y_1 y_\eta\} \cup \{x_1 y_2, x_1 y_\eta, y_1 x_2, y_1 x_\eta\}$.

$p = 2\eta$ and $q = 2\eta + 4$.

To define $\sigma : V(G) \to [0,1]$ the following cases are to be considered.

Case 1: $\eta \equiv 0(mod\ 6)$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 2,3(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(x_\iota) = 0.2 \quad \iota \equiv 0,5(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(x_\iota) = 0.3 \quad \iota \equiv 1,4(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(y_\iota) = 0.1 \quad \iota \equiv 0,1(mod\ 6) \quad 1 \leq \iota \leq \eta - 2$
$\sigma(y_\iota) = 0.2 \quad \iota \equiv 3,4(mod\ 6) \quad 1 \leq \iota \leq \eta - 2$
$\sigma(y_\iota) = 0.3 \quad \iota \equiv 2,5(mod\ 6) \quad 1 \leq \iota \leq \eta - 2$
$\sigma(y_{\eta-1}) = 0.1, \sigma(y_\eta) = 0.3$

Case 2: $\eta \equiv 1(mod\ 6)$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 3,4(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(x_\iota) = 0.2 \quad \iota \equiv 0,1(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(x_\iota) = 0.3 \quad \iota \equiv 2,5(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(y_1) = 0.1$
$\sigma(y_\iota) = 0.1 \quad \iota \equiv 0,5(mod\ 6) \quad 2 \leq \iota \leq \eta$
$\sigma(y_\iota) = 0.2 \quad \iota \equiv 2,3(mod\ 6) \quad 2 \leq \iota \leq \eta$
$\sigma(y_\iota) = 0.3 \quad \iota \equiv 1,4(mod\ 6) \quad 2 \leq \iota \leq \eta$

Case 3: $\eta \equiv 2(mod\ 6)$

The labeling of x_ι for $1 \leq \iota \leq \eta$is same as in Case 1. The labeling of y_ι for $1 \leq \iota \leq \eta - 2$ same as in Case 1 and $\sigma(y_{\eta-1}) = 0.2, \sigma(y_\eta) = 0.2$

Case 4: $\eta \equiv 3(mod\ 6)$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 0,1(mod\ 6) \quad 1 \leq \iota \leq \eta - 2$
$\sigma(x_\iota) = 0.2 \quad \iota \equiv 3,4(mod\ 6) \quad 1 \leq \iota \leq \eta - 2$
$\sigma(x_\iota) = 0.3 \quad \iota \equiv 2,5(mod\ 6) \quad 1 \leq \iota \leq \eta - 2$
$\sigma(x_{\eta-1}) = 0.3, \sigma(x_\eta) = 0.3$
$\sigma(y_\iota) = 0.1 \quad \iota \equiv 1,2(mod\ 6) \quad 1 \leq \iota \leq \eta - 2$
$\sigma(y_\iota) = 0.2 \quad \iota \equiv 4,5(mod\ 6) \quad 1 \leq \iota \leq \eta - 2$
$\sigma(y_\iota) = 0.3 \quad \iota \equiv 0,3(mod\ 6) \quad 1 \leq \iota \leq \eta - 2$
$\sigma(y_{\eta-1}) = 0.2, \sigma(y_\eta) = 0.2$

Case 5: $\eta \equiv 4(mod\ 6)$

The labeling of x_i for $1 \leq \iota \leq \eta - 4$ is same as in Case 4. $\sigma(x_{\eta-3}) = 0.2, \sigma(x_{\eta-2}) = 0.3, \sigma(x_{\eta-1}) = 0.2, \sigma(x_\eta) = 0.1$
$\sigma(y_\iota) = 0.1 \quad \iota \equiv 2,3(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(y_\iota) = 0.2 \quad \iota \equiv 0,5(mod\ 6) \quad 1 \leq \iota \leq \eta$
$\sigma(y_\iota) = 0.3 \quad \iota \equiv 1,4(mod\ 6) \quad 1 \leq \iota \leq \eta$

Case 6: $\eta \equiv 5(mod\ 6)$

The labeling of x_ι for $1 \leq \iota \leq \eta - 2$ is same as in Case 4. $\sigma(x_{\eta-1}) = 0.2, \sigma(x_\eta) = 0.3$

The labeling of y_ι for $1 \leq \iota \leq \eta$ is same as in Case 5.

The following table gives the number of vertices and edges which are labelled with $k \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$

Table 9: -$v_\sigma(i)$ for mutual duplication of a pair of vertices intwo copies of cycle C_n

Nature of η	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$
$\eta \equiv 0(mod\ 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$
$\eta \equiv 1(mod\ 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$
$\eta \equiv 2(mod\ 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$
$\eta \equiv 3mod6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$
$\eta \equiv 4(mod\ 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$	$\frac{p+1}{3}$
$\eta \equiv 5(mod\ 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$

Table 10: -$e_\mu(i)$ for mutual duplication of a pair of vertices intwo copies of cycle C_n

Nature of η	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$\eta \equiv 0(mod\ 6)$	$\frac{q-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$
$\eta \equiv 1(mod\ 6)$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$\eta \equiv 2(mod\ 6)$	$\frac{q+1}{3} - 1$	$\frac{q+1}{3}$	$\frac{q+1}{3}$
$\eta \equiv 3mod6)$	$\frac{q-1}{3} + 1$	$\frac{q-1}{3}$	$\frac{q-1}{3}$
$\eta \equiv 4(mod\ 6)$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$\eta \equiv 5(mod\ 6)$	$\frac{q+1}{3} - 1$	$\frac{q+1}{3}$	$\frac{q+1}{3}$

From the above table 9 and 10 , we can easily find that the conditions of fuzzy quotient - 3 cordial labeling, i.e., $|v_\sigma(i) - v_\sigma(j)| \leq 1$ and $|e_\mu(i) - e_\mu(j)| \leq 1$ for $i \neq j$, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ is satisfied. Hence the graph obtained by the mutual duplication of a pair of vertices in two copies of cycle C_n admits fuzzy quotient - 3 cordial labeling.

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Theorem 3.6 The graph obtained by the mutual duplication of a pair of edges in two copies of cycle C_η admits fuzzy quotient - 3 cordial labeling.

Proof:

Let G be the graph obtained by the mutual duplication of a pair of edges each respectively from each copy of cycle C_η. Without loss of generality assume that the edge be x_1x_n from the first copy of cycle C_n and the edge be y_1y_n from the second copy of cycle C_η are mutually duplicated.

$V(G) = \{x_\iota : 1 \leq \iota \leq \eta\} \cup \{y_\iota : 1 \leq \iota \leq \eta\}$ and $E(G) = \{x_\iota x_{\iota+1} : 1 \leq \iota \leq \eta - 1\} \cup \{x_1 x_\eta\} \cup \{y_\iota y_{\iota+1} : 1 \leq \iota \leq \eta -$ *1∪y1y η∪x1y2,x2y1,xη−1yη,xηyη−1.*

$p = 2\eta$ and $q = 2\eta + 4$.. To define $\sigma: V(G) \to [0,1]$ the following cases are to be considered.

Case 1:$\eta \equiv 0(mod\ 6)$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 1,2(mod\ 6) \quad 1 \leq \iota \leq \eta$

$\sigma(x_\iota) = 0.2 \quad \iota \equiv 4,5(mod\ 6) \quad 1 \leq \iota \leq \eta$

$\sigma(x_\iota) = 0.3 \quad \iota \equiv 0,3(mod\ 6) \quad 1 \leq \iota \leq \eta$

$\sigma(y_\iota) = 0.1 \quad \iota \equiv 0,1(mod\ 6) \quad 1 \leq \iota \leq \eta$

$\sigma(y_\iota) = 0.2 \quad \iota \equiv 3,4(mod\ 6) \quad 1 \leq \iota \leq \eta$

$\sigma(y_\iota) = 0.3 \quad \iota \equiv 2,5(mod\ 6) \quad 1 \leq \iota \leq \eta$

Case 2:$\eta \equiv 1(mod\ 6)$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 0,5(mod\ 6) \quad 1 \leq \iota \leq \eta - 1$

$\sigma(x_\iota) = 0.2 \quad \iota \equiv 2,3(mod\ 6) \quad 1 \leq \iota \leq \eta - 1$

$\sigma(x_\iota) = 0.3 \quad \iota \equiv 1,4(mod\ 6) \quad 1 \leq \iota \leq \eta - 1$

$\sigma(x_\eta) = 0.1$

$\sigma(y_1) = 0.2$

$\sigma(y_\iota) = 0.1 \quad \iota \equiv 1,2(mod\ 6) \quad 2 \leq \iota \leq \eta$

$\sigma(y_\iota) = 0.2 \quad \iota \equiv 4,5(mod\ 6) \quad 2 \leq \iota \leq \eta$

$\sigma(y_\iota) = 0.3 \quad \iota \equiv 0,3(mod\ 6) \quad 2 \leq \iota \leq \eta$

Case 3:$\eta \equiv 2(mod\ 6)$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 2,3(mod\ 6) \quad 1 \leq \iota \leq \eta$

$\sigma(x_\iota) = 0.2 \quad \iota \equiv 0,5(mod\ 6) \quad 1 \leq \iota \leq \eta$

$\sigma(x_\iota) = 0.3 \quad \iota \equiv 1,4(mod\ 6) \quad 1 \leq \iota \leq \eta$

$y_1 = 0.1$ and the labeling of y_i for $2 \leq \iota \leq \eta - 1$ is same as in Case 2 and $\sigma(y_\eta) = 0.2$

Case 4:$\eta \equiv 3(mod\ 6)$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 0,1(mod\ 6) \quad 1 \leq \iota \leq \eta$

$\sigma(x_\iota) = 0.2 \quad \iota \equiv 3,4(mod\ 6) \quad 1 \leq \iota \leq \eta$

$\sigma(x_\iota) = 0.3 \quad \iota \equiv 2,5(mod\ 6) \quad 1 \leq \iota \leq \eta$

The labeling of y_ι for $1 \leq \iota \leq \eta - 2$ is same as in Case 1 and $\sigma(y_{\eta-1}) = 0.2$, $\sigma(y_\eta) = 0.3$

Case 5:$\eta \equiv 4(mod\ 6)$

The labeling of x_ι for $1 \leq \iota \leq \eta - 1$ is same as in Case 4 $y_1 = 0.1$ and the labeling of y_ι is same as in Case 2 for $2 \leq \iota \leq \eta$

Case 6:$\eta \equiv 5(mod\ 6)$

The labeling of x_ι for $1 \leq \iota \leq \eta - 1$ is same as in Case 5 and $\sigma(x_\eta) = 0.3$

$y_1 = 0.1$ and the labeling of y_ι is same as in Case 2 for $2 \leq \iota \leq \eta$

The following table gives the number of vertices and edges which are labelled with $k \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$

Table 11: - $v_\sigma(i)$ for mutual duplication of a pair of edgesin two copies of cycle C_n

Nature of η	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$
$\eta \equiv 0(mod\ 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$
$\eta \equiv 1(mod\ 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$
$\eta \equiv 2(mod\ 6)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$
$\eta \equiv 3mod6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$
$\eta \equiv 4(mod\ 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$	$\frac{p+1}{3}$
$\eta \equiv 5(mod\ 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$

Table 12: - $e_\mu(i)$ for mutual duplication of a pair of edgesin two copies of cycle C_n

Nature of η	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$\eta \equiv 0(mod\ 6)$	$\frac{q-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$
$\eta \equiv 1(mod\ 6)$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$\eta \equiv 2(mod\ 6)$	$\frac{q+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3} - 1$
$\eta \equiv 3mod6)$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$	$\frac{q-1}{3}$
$\eta \equiv 4(mod\ 6)$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$\eta \equiv 5(mod\ 6)$	$\frac{q+1}{3} - 1$	$\frac{q+1}{3}$	$\frac{q+1}{3}$

From the above table 11 and 12 , we can easily find that the conditions of fuzzy quotient - 3 cordial labeling, i.e., $|v_\sigma(i) - v_\sigma(j)| \leq 1$ and $|e_\mu(i) - e_\mu(j)| \leq 1$ for $i \neq j$, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ is satisfied. Hence the graph obtained by the mutual duplication of a pair of edges in two copies of cycle $C_n$admits fuzzy quotient - 3 cordial labeling.

Theorem 3.7 Vertex switching of cycle C_η admits fuzzy quotient-3 cordial labeling.

Proof:

Let $x_\iota : 1 \leq \iota \leq \eta$ be the vertices of the cycle C_η and G be the graph obtained by switching the vertex x_1 of C_n.

$V(G) = \{x_\iota : 1 \leq \iota \leq \eta\}$ and $E(G) = \{x_\iota x_{\iota+1} : 2 \leq \iota \leq \eta - 1\} \cup \{x_1 x_\iota : 3 \leq \iota \leq \eta - 1\}$.

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$p = n$ and $q = 2\eta - 5$.

To define $\sigma: V(G) \to [0,1]$ the following cases are to be considered.

Case 1:$\eta \equiv 0,2,5(mod\ 6)$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 2,3(mod\ 6) \quad 1 \le \iota \le \eta$

$\sigma(x_\iota) = 0.2 \quad \iota \equiv 0,5(mod\ 6) \quad 1 \le \iota \le \eta$

$\sigma(x_\iota) = 0.3 \quad \iota \equiv 1,4(mod\ 6) \quad 1 \le \iota \le \eta$

Case 2:$\eta \equiv 1(mod\ 6)$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 0,5(mod\ 6) \quad 1 \le \iota \le \eta - 1$

$\sigma(x_\iota) = 0.2 \quad \iota \equiv 2,3(mod\ 6) \quad 1 \le \iota \le \eta - 1$

$\sigma(x_\iota) = 0.3 \quad \iota \equiv 1,4(mod\ 6) \quad 1 \le \iota \le \eta - 1$

$\sigma(x_\eta) = 0.2$

Case 3:$\eta \equiv 3(mod\ 6)$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 0,1(mod\ 6) \quad 1 \le \iota \le \eta$

$\sigma(x_\iota) = 0.2 \quad \iota \equiv 3,4(mod\ 6) \quad 1 \le \iota \le \eta$

$\sigma(x_\iota) = 0.3 \quad \iota \equiv 2,5(mod\ 6) \quad 1 \le \iota \le \eta$

Case 4:$\eta \equiv 4(mod\ 6)$

$\sigma(x_1) = 0.3$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 4,5(mod\ 6) \quad 2 \le \iota \le \eta$

$\sigma(x_\iota) = 0.2 \quad \iota \equiv 1,2(mod\ 6) \quad 2 \le \iota \le \eta$

$\sigma(x_\iota) = 0.3 \quad \iota \equiv 0,3(mod\ 6) \quad 2 \le \iota \le \eta$

The following table gives the number of vertices and edges which are labelled with $k \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$

Table 13: -$v_\sigma(i)$ for the vertex switching of a cycle C_n

Nature of η	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$
$\eta \equiv 0(mod\ 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$
$\eta \equiv 1(mod\ 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3}+1$	$\frac{p-1}{3}$
$\eta \equiv 2(mod\ 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3}-1$	$\frac{p+1}{3}$
$\eta \equiv 3(mod\ 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$
$\eta \equiv 4(mod\ 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{p-1}{3}+1$
$\eta \equiv 5(mod\ 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3}-1$	$\frac{p+1}{3}$

Table 14: -$e_\mu(i)$ for the vertex switching of a cycle C_n

Nature of η	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$\eta \equiv 0(mod\ 6)$	$\frac{q-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3}+1$
$\eta \equiv 1(mod\ 6)$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$\eta \equiv 2(mod\ 6)$	$\frac{q+1}{3}-1$	$\frac{q+1}{3}$	$\frac{q+1}{3}$
$\eta \equiv 3(mod\ 6)$	$\frac{q-1}{3}$	$\frac{q-1}{3}+1$	$\frac{q-1}{3}$
$\eta \equiv 4(mod\ 6)$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$\eta \equiv 5(mod\ 6)$	$\frac{q+1}{3}$	$\frac{q+1}{3}-1$	$\frac{q+1}{3}$

From the above table 13 and 14 , we can easily find that the conditions of fuzzy quotient - 3 cordial labeling, i.e., $|v_\sigma(i) - v_\sigma(j)| \le 1$ and $|e_\mu(i) - e_\mu(j)| \le 1$ for $i \ne j$, where $i,j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ is satisfied. Hence the vertex switching of cycle C_n admits fuzzy quotient - 3 cordial labeling.

Theorem 3.8 The joint sum of two copies of cycle C_η admits fuzzy quotient-3 cordial labeling.

Proof:

Let we denote the vertices of first copy of C_η by $x_\iota: 1 \le \iota \le \eta$ and vertices of second copy by $x_{\eta+\iota}: 1 \le \iota \le \eta$ and G be the resultant graph obtained by joing the two copies of C_η with a new edge $x_1 x_{\eta+1}$.

Then vertex set $V(G) = \{x_\iota: 1 \le \iota \le 2\eta\}$ and $E(G) = \{x_\iota x_{\iota+1}: 1 \le \iota \le \eta - 1\} \cup \{x_1 x_\eta\} \cup \{x_1 x_{\eta+1}\} \cup \{x_\iota x_{\iota+1}: \eta + 1 \le \iota \le 2\eta - 1\} \cup \{x_{\eta+1} x_{2\eta}\}$.

$p = 2\eta$ and $q = 2\eta + 1$.

To define $\sigma: V(G) \to [0,1]$ the following cases are to be considered.

Case 1:$\eta \equiv 0,2,3,5(mod\ 6)$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 1,2(mod\ 6) \quad 1 \le \iota \le 2\eta$

$\sigma(x_\iota) = 0.2 \quad \iota \equiv 4,5(mod\ 6) \quad 1 \le \iota \le 2\eta$

$\sigma(x_\iota) = 0.3 \quad \iota \equiv 0,3(mod\ 6) \quad 1 \le \iota \le 2\eta$

Case 2:$\eta \equiv 1(mod\ 6)$

$\sigma(x_\iota) = 0.1 \quad \iota \equiv 4,5(mod\ 6) \quad 1 \le \iota \le \eta$

$\sigma(x_\iota) = 0.2 \quad \iota \equiv 1,2(mod\ 6) \quad 1 \le \iota \le \eta$

$\sigma(x_\iota) = 0.3 \quad \iota \equiv 0,3(mod\ 6) \quad 1 \le \iota \le \eta$

$\sigma(x_{\eta+1}) = 0.1, \ \sigma(x_{\eta+2}) = 0.1, \ \sigma(x_{\eta+3}) = 0.2, \ \sigma(x_{\eta+4}) = 0.3$

$\sigma(x_{\eta+i}) = 0.1 \iota \equiv 2,3(mod\ 6) \quad 5 \le \iota \le 2\eta$

$\sigma(x_{\eta+i}) = 0.2 \iota \equiv 0,5(mod\ 6) \quad 5 \le \iota \le 2\eta$

$\sigma(x_{\eta+i}) = 0.3 \iota \equiv 1,4(mod\ 6) \quad 5 \le \iota \le 2\eta$

Case 3:$\eta \equiv 4(mod\ 6)$

The labeling of x_i , $x_{\eta+i}$ for $1 \le i \le \eta - 1$ is same as in Case 1 and $\sigma(x_\eta) = 0.3, \sigma(x_{2\eta}) = 0.2$

The following table gives the number of vertices and edges which are labelled with $k \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$

Table 15: - $v_\sigma(i)$ for the joint sum of two copies of cycle C_n

Nature of η	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$
$\eta \equiv 0(mod\ 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$

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$\eta \equiv 1(mod\ 6)$	$\frac{p+1}{3}-1$	$\frac{p+1}{3}$	$\frac{p+1}{3}$
$\eta \equiv 2(mod\ 6)$	$\frac{p-1}{3}+1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$
$\eta \equiv 3(mod\ 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$
$\eta \equiv 4(mod\ 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3}-1$	$\frac{p+1}{3}$
$\eta \equiv 5(mod\ 6)$	$\frac{p-1}{3}+1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$

Table 16 - $e_\mu(i)$ for the joint sum of two copies of cycle C_n

Nature of η	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$\eta \equiv 0(mod\ 6)$	$\frac{q-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3}+1$
$\eta \equiv 1(mod\ 6)$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$\eta \equiv 2(mod\ 6)$	$\frac{q+1}{3}-1$	$\frac{q+1}{3}$	$\frac{q+1}{3}$
$\eta \equiv 3(mod\ 6)$	$\frac{q-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3}+1$
$\eta \equiv 4(mod\ 6)$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$\eta \equiv 5(mod\ 6)$	$\frac{q+1}{3}$	$\frac{q+1}{3}-1$	$\frac{q+1}{3}$

From the above table 15 and 16 , we can easily find that the conditions of fuzzy quotient - 3 cordial labeling, i.e., $|v_\sigma(i) - v_\sigma(j)| \leq 1$ and $|e_\mu(i) - e_\mu(j)| \leq 1$ for $i \neq j$, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ is satisfied. Hence the joint sum of two copies of cycle C_n admits fuzzy quotient - 3 cordial labeling.

IV. CONCLUSION

Graph labeling is an interesting concept in graph theory. It becomes a significant branch of interdisciplinary research between mathematics and other sciences. Since all the graphs does not admit the fuzzy qoutient-3 cordial labeling, it creates more interest to investigate the graphs of different families. In this work we have discussed and established the existence of fuzzy qoutient-3 cordial labeling in the context of some graph operations on cycle graph. Investigating this labeling concept on other families of graphs will be our future.

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