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Soliton Solution for Nonlinear Partial Differential Equations by Sine-Function Method

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***Abstract* - In this study, we established a traveling wave solution by using Sine-function algorithm for modified Benjamin-Bona-Mahony equation.**

Keywords: Sine-function method, modified Benjamin-Bona-Mahony equation, solution solutions.

I. INTRODUCTION

The study of numerical methods for the solution of nonlinear partial differential equations has enjoyed an intense period of activity over the last 50 years from both theoretical and practical points of view. Improvements in numerical techniques, together with the rapid advances in computer technology, have meant that many of the partial differential equations arising from engineering and scientific applications, which were previously intractable, can now, be routinely solved [1]. Finite difference methods approximate the differential operators and hence, solve the difference equations. In finite element method the continuous domain is represented as a collection of a finite number N of sub domains known as elements. The collection of elements is called the finite element mesh. For time dependent problems, the differential equations are approximated by the finite element method to obtain a set of ordinary differential equations in time. These differential equations are solved approximately by finite difference methods or other methods. In all finite difference and finite elements it is necessary to have a boundary and initial conditions. But the Adomian decomposition method [2], Depends only on the initial conditions to obtain solution in series form which almost converges to the exact solutions of the problem. In recent years, some other methods have been developed, such as, the tanh method [3,4], the extended tanh-function method [5,6], the modified extended tanh-function method [7-11], variational iteration method [12-17], the generalized hyperbolic-function [18,19], the separation of variables method [20.21], the sine–cosine method [22,23] and the cosine-function method [24].

The purpose of this paper is to extend the sine function method to find the exact soliton solutions of the important nonlinear partial differential equation modified Benjamin-Bona-Mahony equation.

II. THE SINE FUNCTION METHOD

Consider the nonlinear partial differential equation of the form

$$F(u, u_t, u_x, u_{xx}, u_{xxt}, \ldots) = 0 \qquad (1)$$

Where $u(x,t)$ is the solution of nonlinear partial differential equation (1). We use the transformations,

$$u(x,t) = f(\xi), \quad \xi = x - ct \qquad (2)$$

This enables us to use the following changes:

$$\frac{\partial}{\partial t}(\cdot) = -c\frac{d}{d\xi}(\cdot), \quad \frac{\partial}{\partial x}(\cdot) = \frac{d}{d\xi}(\cdot), \quad \frac{\partial^2}{\partial x^2}(\cdot) = \frac{d^2}{d\xi^2}(\cdot), \ldots \qquad (3)$$

Eq. (3) changes Eq. (1) in the form

$$G(f, f', f'', f''', \ldots) = 0 \qquad (4)$$

The solution of Eq. (4) can be expressed in the form:

$$f(\xi) = \lambda \sin^{\alpha}(\mu\xi), \qquad |\xi| \le \frac{\pi}{\mu} \qquad (5)$$

Where λ, α and μ are unknown parameters which are to be determined. Thus we have:

$$f' = \frac{df(\xi)}{d\xi} = \lambda\,\alpha\,\mu\,\sin^{\alpha-1}(\mu\xi)\cos(\mu\xi) \qquad (6)$$

$$f'' = \frac{d^2 f(\xi)}{d\xi^2} = -\lambda\mu^2\alpha\sin^{\alpha}(\mu\xi) + \lambda\mu^2\alpha(\alpha-1)\sin^{\alpha-2}(\mu\xi) - \lambda\mu^2\alpha(\alpha-1)\sin^{\alpha}(\mu\xi) \qquad (7)$$

:

:

Substituting Eq. (5) in Eq. (4) gives a trigonometric equation of $\sin^{\alpha}(\mu\xi)$ terms. To determine the parameters first balancing the exponents of each pair of sine to find α. Then collecting all terms with the same power in $\sin^{\alpha}(\mu\xi)$ and put to zero their coefficients to get a system of algebraic

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equations among the unknowns λ, α and μ. Now the problem is reduced to a system of algebraic equations that can be solved to obtain the unknown parameters λ, α and μ. Hence, the solution considered in Eq. (5) is obtained. The above analysis yields the following theorem:

Theorem: The exact analytical solution of the nonlinear partial differential equations Eq. (1) can be determined in the form Eq. (5) where all constants found from the algebraic equations after its solutions.

III. APPLICATION

In order to illustrate the effectiveness of the proposed method we will consider the modified Benjamin-Bona-Mahony equation [25-27] in the following form:

$$u_t + u_x + au^2 u_x + bu_{xxt} = 0 \qquad (8)$$

Where a and b are constants.

Using the transformation, $u(x,t) = f(\xi), \quad \xi = x - ct$, Eq. (8) reduces to:

$$(1-c)f' + a\, f^2\, f' - bcf''' = 0 \qquad (9)$$

Integrating Eq. (9) with respect to ξ and considering the constants for integration to be zeros, we receive

$$(1-c)f + \frac{a}{3} f^3 - bcf'' = 0 \qquad (10)$$

Substituting Eq. (5) and (7) into (10) gives:

$$(1-c)\lambda \sin^{\alpha}(\mu\,\xi) + \frac{a}{3}\lambda^3 \sin^{3\alpha}(\mu\,\xi) + bc\lambda\alpha\mu^2 \sin^{\alpha}(\mu\,\xi) - bc\lambda\mu^2\alpha(\alpha-1)\sin^{\alpha-2}(\mu\,\xi)$$

$$+bc\lambda\mu^2\alpha(\alpha-1)\sin^{\alpha}(\mu\,\xi) = 0 \qquad (11)$$

Eq. (11) is satisfied only if the following system of algebraic equations holds:

$$3\alpha = \alpha - 2,$$

$$(1-c)\lambda + bc\lambda\alpha\mu^2 + bc\lambda\mu^2\alpha(\alpha-1) = 0,$$

$$\frac{a}{3}\lambda^3 - bc\lambda\mu^2\alpha(\alpha-1) = 0. \qquad (12)$$

Solving the system of equations (12), we obtain:

$$\alpha = -1, \quad \mu = \pm\sqrt{\frac{c-1}{bc}}\ , \quad and \quad \lambda = \pm\sqrt{\frac{6(c-1)}{a}} \qquad (13)$$

Substituting Eq. (13) into Eq. (5) we obtain the exact soliton solution of the modified Benjamin-Bona-Mahony equation in the form

$$u(x,t) = \pm\sqrt{\frac{6(c-1)}{a}}\, \sin^{-1}\left(\pm\sqrt{\frac{c-1}{bc}}\,(x-ct)\right) \qquad (14)$$

This gives the desired exact soliton solution of the modified Benjamin-Bona-Mahony equation.

IV. CONCLUSION

In this paper, the sine-function method has been successfully applied to find the solution for modified Benjamin-Bona-Mahony equation. The sine-function method is used to find new exact solution. Thus, it is possible that the proposed method can be extended to solve the problems of nonlinear partial differential equations which arising in the theory of solitons and other areas.

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