

Design Robust Controllers of Convey-Crane System under Different Types of Uncertainty

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Abstract - In this paper, the designing of robust controllers for well-known convey crane systems in the presence of uncertainties is presented. Cranes play a great role in the industrial environment, construction sites, ship-yard, and rial-yard because of their efficiency and accuracy in moving heavy and dangerous materials from one loading location to another. The main challenge in controlling such kinds of systems is to minimize the undesirable swings when different payloads are carried. This phenomenon is because the convey crane is classified as an under actuated nonlinear system and there are many sources of uncertainties that affect the plant of the physical system Hence, the stability of the convey crane system in the presence of the uncertainties became a challenging task between the researchers in the control engineering field and used as a benchmark in universities. In this research, firstly, the stability of the linearized model of the convey-crane system is analyzed using the edge theorem in the presence of two uncertain parameters: the rope length and the weight of the load, where the classical pole placement technique is used for stabilization purposes. Secondly, the sliding mode control (SMC) is proposed in order to stabilize the nonlinear model of the convey-crane system with the presence of noise signals. In order to test the effectiveness of the proposed SMC controller, a conventional PID control is applied to the linearized model of the convey-crane system, where the PID parameters are tuned using the well-known Ziegler-Nicholas method. The robustness against the uncertainties in the parameters of the convey crane is achieved by using the Edge theorem, whereas the robustness against the noise signal is tested by using MATLAB; the simulation results show the superiority of the SMC controller over the PID.

Keywords: Crane, Slide Mode Control, Robust, PID, Uncertain, Robust control.

I. INTRODUCTION

A convey-crane system is an industrial machine used for lifting and moving heavy materials in a precise way at work places [1]. The convey-crane is widely used in ports and workplaces due to its ease of use, and its ability to save time

and effort [3]. Despite the advantages of convey-cranes, there is a challenge in controlling this type of system, which is reducing unwanted swing when transporting different loads. This phenomenon occurs because the convey-crane is classified as an under actuated nonlinear system [27]. In addition, the failure to control the swing angles leads to difficulties in automating the system for workers. This, in turn, increases the possibility of damaging the quality of the load or the operating environment around the construction site. [10]. Also, in the convey-crane system: rope length uncertainty and load weight uncertainty. These uncertainties have a significant impact on the stability of the system and make the control of the system more challenging. The authors in [7] conducted a comparison between a fuzzy control system and a Linear Quadratic Gaussian control (LQG) system for an overhead crane. They evaluated the applicability of the control algorithms in real-time and assumed that the model represents the actual system. The study included various perturbations to test the robustness of the control algorithms and presented a complete reference trajectory model. In [12], the authors studied the stability of the convey crane system based on the concepts of passivity and Lyapunov. In [3] the authors described the crane system when there are two uncertainty parameters and linearized the nonlinear crane system to apply the control method using the state feedback method. In [16] a control scheme was proposed based on the linear dynamic model of the stable equilibrium of the crane system, which included rapid damping for swing using the loop shaping and root locus methods.



Figure 1: Convey crane

The researchers mainly relied on linearizing the nonlinear system before designing the control law. In [24] a continuous global sliding mode controller was proposed along with a nonlinear disturbance observer for the regulation and disturbance estimation control of the overhead crane system. In [26] a nonlinear control scheme incorporating a parameter adaptive mechanism was devised to ensure the overall closed-loop system stability, with stability proof of the overall system given in terms of the Lyapunov concept. In [22] the authors focused on the design of robust nonlinear controllers based on both conventional and hierarchical sliding mode techniques for double-pendulum overhead crane systems.

The main contribution of this research is to propose a controller for a convey crane system that can handle both parameter uncertainties and noise signal uncertainties. To address parameter uncertainty, the pole placement method was used to design the controller while assuming that the uncertain parameters were fixed at their minimum limit. The stability of the designed controller was then tested using edge theory, which is a method for determining the stability of systems under uncertainty. To handle noise signal uncertainty, a sliding mode controller was proposed as it is known to be robust to noise and disturbances. Additionally, a PID controller was also designed for comparison purposes. The performance of both controllers was then evaluated using simulations to determine which controller is better suited for the convey crane system. Overall, this research is significant as it provides a practical solution for controlling a convey crane system under uncertain conditions.

In Section one, a general introduction to the convey crane system is provided, and previous studies in this field are reviewed. In Section two, a mathematical model is analyzed for the crane system, and the nonlinear model is linearized. In Section three, the effect of parameter uncertainty is studied, and the classification of uncertain parameters is explained. The stability of the convey crane system is analyzed under the presence of two uncertain parameters, and a controller is designed using pole placement methods. In Section four, a sliding mode controller (SMC) is proposed to stabilize the convey crane system.

II. CONVEY CRANE SYSTEM MODEL

The mechanical model of the crane system is illustrated in Figure 2 as shown; the system consists of two main components: the cart and the rope. The crane system has two degrees of freedom and only one control actuator, making it an under actuated system. Assuming that the input signal is the force u , the rope has a mass of m_c , and the rope length parameters are l . The load mass is m_l , and the gravity

acceleration is denoted by g . By writing equation of motion and taking the following into account:

- Dynamics and non-linearity of the driving motor are neglected.
- The crab moves along the track without friction or slip.
- The rope has no mass and no elasticity.
- The input signal is the force u , is the accelerates of rope, the crab mass is m_c , rope length l , load mass m_l , and gravity acceleration g .

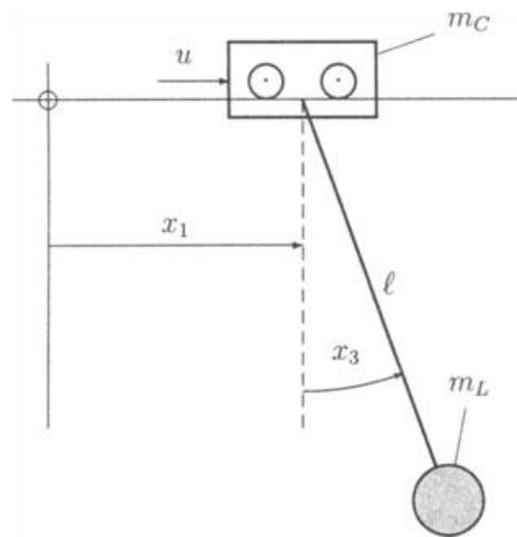


Figure 1: Convey-Crane system mechanical model.

$$T \sin(\theta) = m_l \ddot{x} - m_l L \ddot{\theta} \cos(\theta) + m_l L \dot{\theta}^2 \sin(\theta) \quad (1)$$

$$-m_l g \sin(\theta) = m_l \ddot{x} \cos(\theta) - m_l L \ddot{\theta} \cos(\theta) \quad (2)$$

Define the state vector as follows:

$$x = \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \text{cart - position} \\ \text{cart - velocity} \\ \text{rope - angle} \\ \text{rope - anglerate} \end{bmatrix} \quad (3)$$

So, the nonlinear second order differential equations can be rewritten as the following:

$$[m_l + m_c] \ddot{x}_1 + [m_l L \cos(x_3)] \ddot{x}_3 = m_l L \dot{x}_3^2 \sin(x_3) + u \quad (4)$$

$$[m_l L] \ddot{x}_3 + [m_l L \cos(x_3)] \ddot{x}_1 = -m_l L g \sin(x_3) \quad (5)$$

The equation of motion can be writing as the following matrix:

$$\begin{bmatrix} m_l + m_c & m_l L \cos(x_3) \\ m_l L \cos(x_3) & m_l L \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{bmatrix} = \begin{bmatrix} \dot{x}_3^2 \sin(x_3) + u \\ -m_l g \sin(x_3) \end{bmatrix} \quad (6)$$

In order to use conventional PID control and state feedback control, the nonlinear system must be linearized. To obtain the transfer function of the linearized convey crane system and its characteristic polynomial, the nonlinear equations of motion for the system can be linearized around an operating point. This involves finding the partial derivatives of the equations of motion with respect to the system variables (position, velocity, and control input), evaluating them at the operating point, and using them to construct a linear system of equations.

For small deflection angle x_3 and small angular velocity x_4 :

$$\cos(x_3) \cong 1 \quad \sin(x_3) \cong x_3 \quad \cos(x_3)^2 \cong 0 \quad x_4^2 = 0 \quad (7)$$

By using these assumptions, and assuming $m_l = 0.5 m_c = 0.5$, $L = 1$, and $g = 9.8$ the system can be written as following:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -9.8 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -19.6 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -2 \end{bmatrix} \quad (8)$$

After obtaining the linear system of equations, it can be expressed in state-space form:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (9)$$

$$y(t) = Cx(t) + D(t) \quad (10)$$

Where x is the state vector, u is the input vector, y is the output vector, A is the system matrix, B is the input matrix, C is the output matrix, and D is the feed-through matrix.

The transfer function of the system can then be obtained using the Laplace transform. If we assume that the initial conditions are zero, then the transfer function from input u to output y is given by:

$$G(s) = C(sI - A)^{-1}B + D \quad (11)$$

The characteristic polynomial of the system is the polynomial whose roots are the eigenvalues of the system matrix A . It can be expressed as:

$$\det(sI - A) = 0 \quad (12)$$

Where \det denotes the determinant.

The steady state matrix for convey crane system is shown in eq Eq. (7), To get the transfer function of the linearized convey crane system and characteristic polynomial of A :

$$P_c(s) = s^2 \left[s^2 + \frac{\left(1 + \frac{m_l}{m_c}\right)g}{L} \right] \quad (13)$$

By solving the equation and finding the pols of the polynomial $s_1, s_2 = 0, s_3, s_4 = + - J(4.42)$.

The open loop transfer function of the system is shown in eq (14):

$$TF = \frac{(s^2 + 96.04)}{s^2(s^2 + 19.6)} \quad (14)$$

By finding the roots of the characteristic polynomial, the stability of the linearized convey crane system can be determined. As shown in Figure 3, the poles and zeros are on the imaginary axis, indicating that the open loop crane system is unstable.

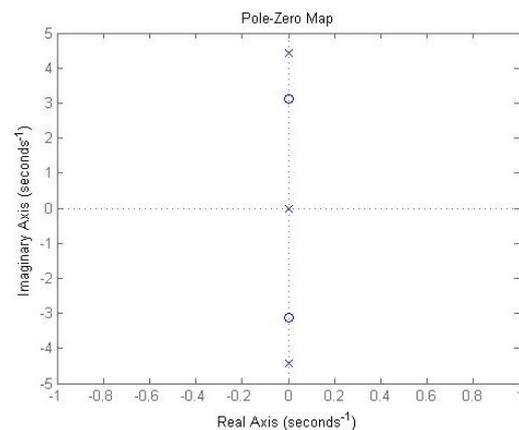


Figure 2: The pols and zeros for the open loop

III. UNCERTAIN PARAMETERS OF CONVEY CRANE SYSTEM

Refer to the variables or factors that have uncertain or unknown values in a system or process. These parameters can affect the behavior and performance of the system, and their values may change over time due to various factors, such as environmental conditions, wear and tear, or changes in the system's configuration. In engineering and control systems, uncertain parameters can have a significant impact on the design and operation of the system, and they need to be carefully considered and accounted for in order to achieve optimal performance and stability. Various techniques and strategies have been developed to address the uncertainty in system parameters, such as robust control, adaptive control, and model predictive control. These techniques aim to improve the system's resilience to changes in the parameters and ensure its stable and efficient operation under various conditions.

3.1 Parameter Box

The parameter box Q is an interval parameter that represents the set of all values that a parameter can take on. For $L=2$, the figure 4 shows the parameter box

In order to analyze the stability of a system under uncertain conditions, it is important to first understand the different types of uncertain systems. According to [3], one way to classify uncertain systems is based on their polynomial expressions and the uncertain parameters that appear within them.

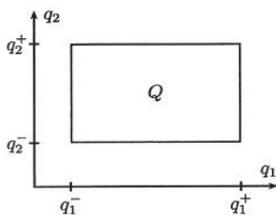


Figure 3: Parameter Box for $L = 2$

3.2 Polynomial Family

Uncertain polynomials can be classified based on how the uncertain parameter is incorporated into the polynomial coefficients. The classification of uncertain polynomials is determined by the following categories:

Interval polynomial: The first type of uncertain polynomial family is known as an interval polynomial. This polynomial has the form:

$$P(s, q) = a_0 + a_1s + a_2s^2 + a_3s^3 + a_4s^4 + \dots + a_n s^n \quad (15)$$

In this expression, P is a continuous function that contains uncertainty, and q is an uncertain coefficient. The coefficients of an interval polynomial are independent of one another, as noted in [3].

Affine linear coefficient: The second type of uncertain polynomial family is known as a Affine linear polynomial with affine parameter dependence, as described in [3]. In this type of polynomial, the coefficients are defined as:

$$a_i = b_i + C_i^T q \quad (16)$$

Multilinear polynomial: the third type of uncertain polynomial family is the multilinear polynomial. This type of polynomial is similar to the previous one but includes multiplicative terms between the uncertain parameters. A multilinear polynomial is classified as such if it is a first-order polynomial with respect to each of the uncertain parameters and includes coefficients with multiplicative terms between

the uncertain parameters. This classification is described in [3].

Polynomial coefficient: The final type of uncertain polynomial family is the polynomial with coefficient dependence. This classification applies when the polynomial is of second-order or higher with respect to each of the uncertain parameters. Such a polynomial is referred to as a polynomial coefficient family. This type of polynomial is the most difficult to analyze for stability, as noted in [3].

3.3 Convey Crane System with uncertain parameters

The crane system involves two uncertain parameters, namely the length of the rope, $L = q_1$, and the weight of the load, $ml = q_2$. When designing the control law for the system, it is important to consider the uncertainties associated with these parameters. The control law should be able to perform its task effectively over the entire range of possible parameter values. These uncertain parameters can be represented as $Q = [q_1, q_2, q_3, \dots, q_L]$, where Q is a vector of uncertain real parameters and L is the number of uncertain parameters. For the convey-crane system, the length of the rope, L , can vary between 0.5 and 3 meters, while the weight of the load, ml , can vary between 1 and 100 kilograms. Therefore, the control law should be designed to ensure stability over this entire range of possible parameter values.

3.4 State feedback control for convey crane system

State feedback involves the use of the state vector to compute the control action for specified system dynamics. figure [5] shows a linear system (A, B, C) with constant state feedback gain matrix K The steady state matrix for convey crane system is shown in eq Eq. (7), using certain parameter for length of the rope and the weight of the load.

In order to design the pole placement. The closed-loop characteristic polynomial is:

$$P(s, q, k) = \det[SI - A(q) + b(q)k^T] \quad (17)$$

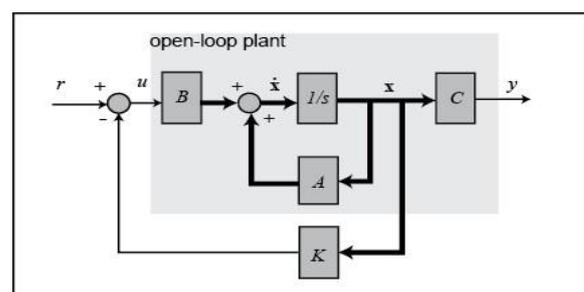


Figure 4: State feedback block diagram

By rewriting Eq. 17:

$$p(s, q, k) = (Lm_c)S^4 + (k_2L - k_4)S^3 + ((m_l + m_c)g + k_1L + k_3)S^2 + (gk_2)S + (gk_1) \quad (18)$$

The coefficients of the close loop characteristic equation must be all positive such that:

$$\begin{aligned} a_0 &= gk_1 > 0 \Rightarrow k_1 > 0 \\ a_1 &= gk_2 > 0 \Rightarrow k_2 > 0 \\ a_2 &= (m_l + m_c)g + k_1L - k_3 > 0 \Rightarrow k_3 < (m_l + m_c)g + k_1L \\ a_3 &= k_2L - k_4 > 0 \Rightarrow k_4 < k_2L \\ a_4 &= Lm_c > 0 \end{aligned} \quad (19)$$

Assuming that, $m_l = 0.5$, $m_c = 0.5$, $L = 1$, and $g = 9.8$ Using the State Feedback method to stabilize the system, the values of the K vector can be assumed as $KT = [1, 1, 1, 0]$. These values satisfy the conditions in eq. [19].

q_1 is the length of the rope and it changes from (0.5-3) m, the q_2 is the mass of the load and it changes from (1-100) K, the gravity acceleration $g = 9.8$ and the cart mass is 30 K. the state feedback control is $KT = [1, 1, 1, 0]$. By substituting the value in the characteristic polynomial:

$$\begin{aligned} a_0 &= 9.8 q_1, a_1 = 9.8 q_2, a_2 \\ &= 9.8 q_1 + q_2 + 293, a_3 \\ &= q_1, a_4 = 30q_1 \end{aligned} \quad (20)$$

Depending on how the uncertain parameter enters into the coefficients of the crane polynomial, the resulting polynomial can be classified as an affine linear polynomial. To investigate the stability of an affine linear polynomial system, the edge theory can be used.

IV. STABILITY OF CONVEY-CRANE WITH THE UNCERTAIN PARAMETERS

An uncertain system is considered stable if and only if all the elements of the set represented by the uncertain system are stable. The characteristic polynomial of the crane system with uncertain parameters, represented by the vector q , is given by the polynomial in 18 with the feedback gain vector K using the state feedback method. This characteristic polynomial belongs to the affine linear polynomial family. As previously mentioned, the proposed convey crane system belongs to the affine linear polynomial family. The edge theorem is one of the theories used to check the stability of systems with uncertain parameters. An affine linear polynomial system is robustly stable if and only if the edge polynomials are robustly stable [3]. The Crane system has two uncertain parameters, namely q_1 ranging from 0.5 to 3 and q_2 ranging from 1 to 100. Therefore, four vertex polynomials can be found, which are

polynomials with fixed parameter values located at the corners of the parameter box shown in Figure 6. These equations are represented as P_{--} , P_{-+} , P_{+-} , and P_{++} .

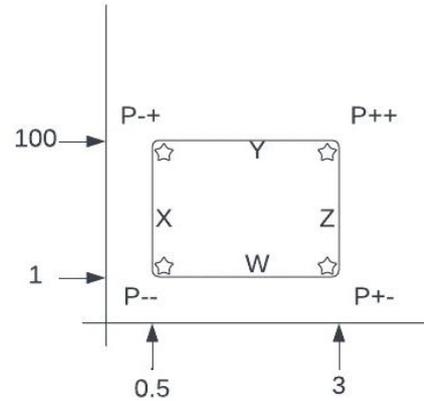


Figure 5: Parameter Box for convey crane system with $L = 2$

The four vertex polynomials can be obtained as shown in equations 21, 22, 23, and 24:

$$P1(s, 0.5, 1) = 15S^4 + 0.5S^3 + 298.9S^2 + 9.8S + 9.8 \quad (21)$$

$$P2(s, 0.5, 100) = 15S^4 + 0.5S^3 + 397.9S^2 + 9.8S + 9.8 \quad (22)$$

$$P3(s, 3, 1) = 90S^4 + 3S^3 + 323.4S^2 + 9.8S + 9.8 \quad (23)$$

$$P4(s, 3, 100) = 90S^4 + 3S^3 + 422.4S^2 + 9.8S + 9.8 \quad (24)$$

The edge polynomials are a set of equations that lie between two vertex polynomials and have one uncertain parameter. As shown in Figure 6, the set of polynomials X lie between two vertex polynomials P_{--} and P_{-+} , while the rest of the edge polynomials are denoted as W , Y , and Z . Thus, the edge polynomials can be written as follows:

$$X = p_{12}(s, \lambda) = (1 - \lambda)p_1(s) + (\lambda)p_2(s), \lambda \in [0, 1] \quad (25)$$

$$Y = p_{24}(s, \lambda) = (1 - \lambda)p_2(s) + (\lambda)p_4(s), \lambda \in [0, 1] \quad (26)$$

$$Z = p_{43}(s, \lambda) = (1 - \lambda)p_4(s) + (\lambda)p_3(s), \lambda \in [0, 1] \quad (27)$$

$$W = p_{31}(s, \lambda) = (1 - \lambda)p_3(s) + (\lambda)p_1(s), \lambda \in [0, 1] \quad (28)$$

The polynomials in Equations 25, 26, 27 and 28 are uncertain polynomials with only one uncertain parameter.

Therefore, Biela’s Theorem can be used to check the stability for a fixed range of the uncertain parameters [3]. By applying the Biela’s Theorem:

Let H_n^b and H_n^c be the Hurwitz matrices of, respectively.

$$Pb(s) = b_0 + b_1s + b_2s^2 + \dots + b_n s^n, b_n \geq 0 \tag{29}$$

$$Pc(s) = c_0 + b_1s + c_2s^2 + \dots + c_n s^n, c_n \geq 0 \tag{30}$$

To determine the robust stability of a polynomial family like the one given in equation 31, we can use the Hurwitz matrices H_n^b and H_n^c for the polynomials $Pb(s)$ and $Pc(s)$, respectively. The system is robustly stable if and only if the following conditions are satisfied:

1-The polynomial $Pb(s)$ is stable, which means that all of its roots have negative real parts.

2-The constant term c_0 of the polynomial $Pc(s)$ is positive, which means that $Pc(0) > 0$. 3-The matrix $(H_n^b)^{-1}H_n^c$ has no nonpositive real eigenvalues. This matrix can be thought of as a transfer function from the input $Pc(s)$ to the output of the Hurwitz matrix for $Pb(s)$. If these conditions are met, the system is guaranteed to be stable for all feedback gains. On the other hand, if any of these conditions are violated, the system may not be robustly stable, even if it appears stable for some feedback gains. [3]:

$$pb(s) = (1 - q)pb(s) + (q)pc(s), q \in [0,1] \tag{31}$$

The first condition is to check the stability of $Pb(s)$ using the Hurwitz table method. After applying this method to the edge polynomials, we can verify that all four edge polynomials are indeed stable.

The second condition in the Biala’s theorem is the $Pc(0) > 0$

$$\begin{aligned} P1(s, 0) &= (9.8) > 0 \\ P2(s, 0) &= (9.8) > 0 \\ P3(s, 0) &= (9.8) > 0 \\ P4(s, 0) &= (9.8) > 0 \end{aligned} \tag{32}$$

The third condition in the Biala’s theorem is the matrix $(H_n^b)^{-1} * H_n^c$ has no unipositive real eigenvalues.

$$(H_3^1) = \begin{bmatrix} 0.5 & 9.8 & 0 \\ 15 & 298.9 & 0 \\ 0 & 0.5 & 9.8 \end{bmatrix} \tag{33}$$

The eigenvalues for the matrix in eq. 33 is shown in eq. 34:

$$eig [H_3^1^{-1} * H_3^2] = \{21.20, 1, 1\} \tag{34}$$

$$(H_3^2) = \begin{bmatrix} 0.5 & 9.8 & 0 \\ 15 & 397.9 & 0 \\ 0 & 0.5 & 9.8 \end{bmatrix} \tag{35}$$

The eigenvalues for the matrix in eq. 35 is shown in eq. 36:

$$eig [H_3^2^{-1} * H_3^4] = \{6, 1.24, 1\} \tag{36}$$

$$(H_3^3) = \begin{bmatrix} 3 & 9.8 & 0 \\ 90 & 323.4 & 0 \\ 0 & 3 & 9.8 \end{bmatrix} \tag{37}$$

The eigenvalues for the matrix in eq. 37 is shown in eq. 38:

$$eig [H_3^3^{-1} * H_3^3] = \{1, 1, 0.227\} \tag{38}$$

$$(H_3^4) = \begin{bmatrix} 3 & 9.8 & 0 \\ 90 & 422.4 & 0 \\ 0 & 3 & 9.8 \end{bmatrix} \tag{39}$$

The eigenvalues for the matrix in eq. 39 is shown in eq. 40:

$$eig [H_3^3^{-1} * H_3^1] = \{1, 0.1667, 0.1667\} \tag{40}$$

As shown above, all real eigenvalues of the Hurwitz matrices are positive. Therefore, based on Biala’s theorem, we can conclude that the vertex polynomials are stable. Furthermore, based on the edge theorem of the affine linear system, we can conclude that the convey-crane system is robustly stable [3].

V. DESIGNING A ROBUST CONTROLLER FOR CONVEY CRANE SYSTEM UNDER NOISE SIGNAL

Designing a robust controller for a convey crane system under noise signal involves developing a controller that can effectively suppress the effects of noise on the system. A robust controller is one that can maintain stable system behavior even when the system is subjected to noise or other disturbances. There are several approaches to designing robust controllers, including H-infinity control and robust control using sliding mode techniques.

For the purpose of comparison, the traditional PID control will be applied to the linearized model of the convey crane system. The PID parameters will be adjusted using the well-known Ziegler-Nichols method.

5.1 Designing PID controller for convey crane system

The traditional PID (proportional-integral-derivative) controller is a widely used feedback control algorithm in industrial and engineering applications. It is a closed-loop

control system that continuously measures the system output and adjusts the system input based on the error between the desired set point and the actual output. The controller works by calculating three terms: the proportional term, the integral term, and the derivative term. The proportional term is proportional to the error, the integral term is proportional to the accumulated error over time, and the derivative term is proportional to the rate of change of the error. The PID controller is widely used due to its simplicity, effectiveness, and ease of implementation.

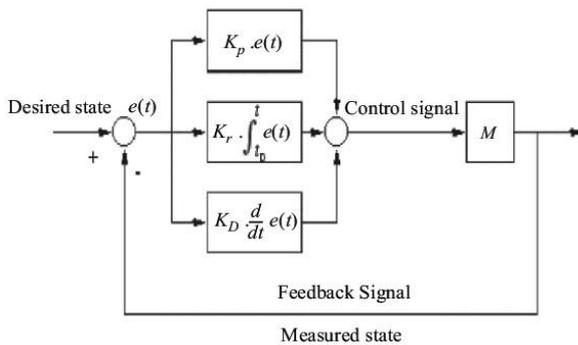


Figure 7: The PID controller block diagram

The Ziegler-Nichols method is a well-known technique for tuning the values of the proportional, integral, and derivative factors in a PID controller. The method involves adjusting the values of these factors based on the system's response to a step change in the set-point. As shown in figure 8 the $t_u = 20.9$. The PID gains are: $Kp = 0.05, Ki = 0.014$, and $Kd = 0.128$ after using Ziegler Nichols technique.

5.2 Simulation Results

To evaluate the proposed conventional PID controller performed, the Simulation was done by using Matlab Simulink.

The physical parameters of the Convey Crane are $m_c = 0.5$ kg, $m_l = 0.5$ kg, $l = 1$ m, and $g = 9.8$ m/s². The design parameters of each control law are chosen as in table 1

The response after applying the PID controller to the linearized convey crane system is stable, and the system reaches the equilibrium point after a finite amount of time, as shown in Figure10. To validate the effectiveness of the PID controller applied to the linearized crane system, it was also applied to the non-linear system. The results are illustrated in Figure 11 to 14.

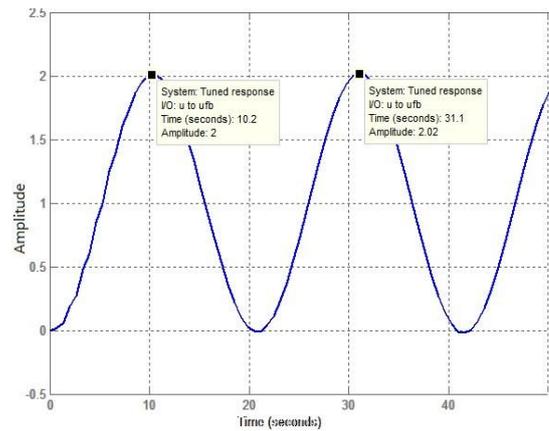


Figure 8: Ziegler Nichols method, T_u

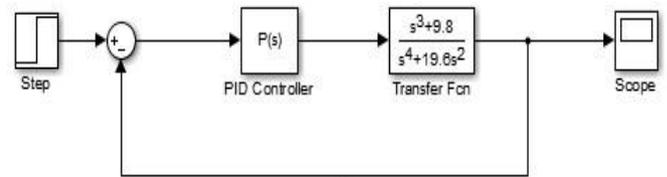


Figure 9: PID controller for the linearized Crane system

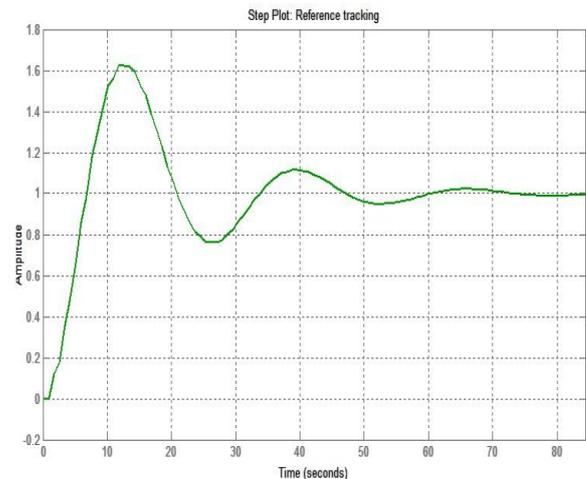


Figure 6: Step response for the linear Crane system

The position state of the convey crane system after applying the PID controller as shown in the figures 11, and it is clear from the figure that the system is unstable.

The cart velocity of the convey crane system after applying the PID controller is shown in Figure 12. It is evident from the figure that the system is unstable.

The rope angle of the convey crane system after applying the PID controller is shown in Figure 14. It is evident from the figure that the system is unstable.

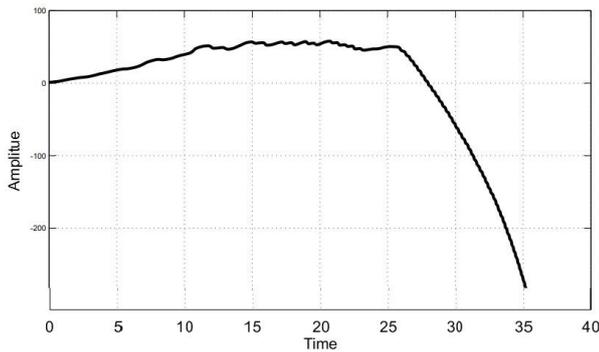


Figure 7: The cart position state of the system

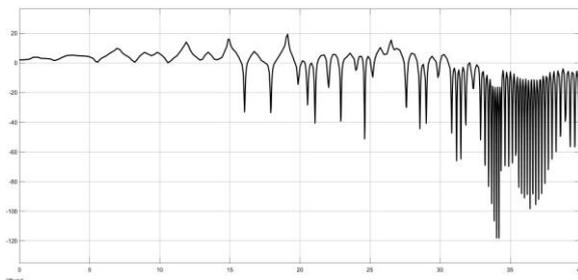


Figure 12: The cart velocity state of the system

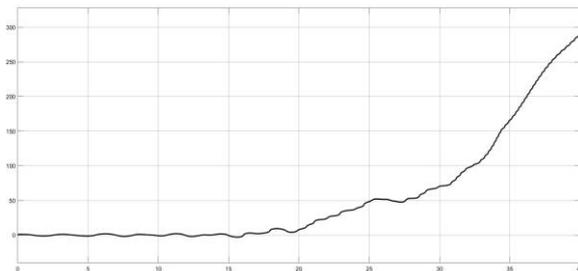


Figure 13: The cart velocity state of the system

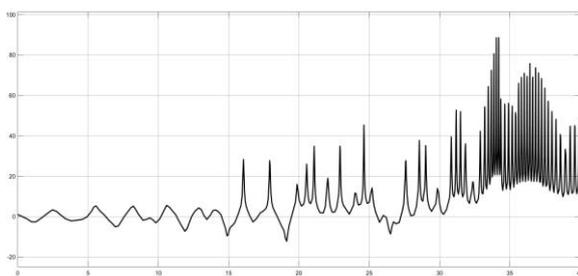


Figure 14: The rope angle rate state of the system

The previous figures demonstrate that the addition of a conventional PID controller to the nonlinear conveyor crane system resulted in system instability, despite the absence of additional noise. To address this issue, a slide mode control method was proposed as an alternative solution for controlling the crane system.

5.3 Designing a Slide Mode Controller for Convey Crane System

Slide mode control is a powerful method for controlling nonlinear systems that may exhibit significant uncertainties

and disturbances. The approach works by defining a specific surface, called the slide surface, in the state space of the system. The control signal is then designed to drive the system towards this surface, where it can slide along the surface towards the desired value. By doing so, the system can achieve fast and robust tracking of the desired output trajectory, even in the presence of external disturbances and modeling uncertainties. Unlike traditional control methods that strive for asymptotic convergence, slide mode control offers finite-time convergence to the desired set-point, which makes it particularly suitable for many practical applications.

The matrix of motion mentioned in equation 6, let:

$$\begin{aligned} G_{11} &= ml + mc, G_{21} = G_{12} \\ &= mL\cos\alpha, G_{22} = mL, f_a \\ &= mlg\sin\alpha, f_u = (x_4)2\sin\alpha \end{aligned} \quad (41)$$

By rewriting the matrix:

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{bmatrix} = \begin{bmatrix} f_a + u \\ f_u \end{bmatrix} \quad (42)$$

Defining the matrix G:

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \quad (43)$$

Then: the system matrix can be written in the following form:

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{bmatrix} = G^{-1} \begin{bmatrix} f_a + u \\ f_u \end{bmatrix} \quad (44)$$

$$\begin{aligned} &G^{-1}(\theta) \begin{bmatrix} f_a + u \\ f_u \end{bmatrix} \\ &= \begin{bmatrix} \frac{-G_{11}}{G_{12}G_{21} - G_{12}G_{22}} & \frac{G_{12}}{G_{12}G_{21} - G_{12}G_{22}} \\ \frac{G_{12}G_{21} - G_{12}G_{22}}{G_{21}} & \frac{-G_{11}}{G_{12}G_{21} - G_{12}G_{22}} \end{bmatrix} * \begin{bmatrix} f_a + u \\ f_u \end{bmatrix} \end{aligned} \quad (45)$$

By solving the equation 44 and 45, the system equations can be rewritten as:

$$\begin{aligned} \ddot{x}_1 &= \frac{-G_{22}f_a}{G_{12}G_{21} - G_{12}G_{22}} + \frac{-G_{22}u}{G_{12}G_{21} - G_{12}G_{22}} \\ &\quad + \frac{G_{12}f_u}{G_{12}G_{21} - G_{12}G_{22}} \end{aligned} \quad (46)$$

$$\ddot{x}_1 = \frac{G_{12}f_u - G_{22}f_a}{G_{12}G_{21} - G_{12}G_{22}} + \frac{-G_{22}}{G_{12}G_{21} - G_{12}G_{22}}u \quad (47)$$

$$\begin{aligned} \ddot{x}_3 &= \frac{-G_{11}f_u}{G_{12}G_{21} - G_{12}G_{22}} + \frac{G_{21}u}{G_{12}G_{21} - G_{12}G_{22}} \\ &\quad + \frac{G_{21}f_a}{G_{12}G_{21} - G_{12}G_{22}} \end{aligned} \quad (48)$$

$$\ddot{x}_3 = \frac{-G_{11}f_u + G_{21}f_a}{G_{12}G_{21} - G_{12}G_{22}} + \frac{G_{21}}{G_{12}G_{21} - G_{12}G_{22}}u \quad (49)$$

The state equation can now be represented as follows:

$$\ddot{x}_1 = f_1 + b_1 u \tag{50}$$

$$\ddot{x}_3 = f_2 + b_2 u \tag{51}$$

Where:

$$f_1 = \frac{G_{12}f_u - G_{22}f_a}{G_{12}G_{21} - G_{12}G_{22}} \tag{52}$$

$$f_2 = \frac{-G_{11}f_u + G_{21}f_a}{G_{12}G_{21} - G_{12}G_{22}} \tag{53}$$

$$b_1 = \frac{-G_{22}}{G_{12}G_{21} - G_{12}G_{22}} \tag{54}$$

$$b_2 = \frac{G_{21}}{G_{12}G_{21} - G_{12}G_{22}} \tag{55}$$

5.4 Slide Surface for convey crane system

The dynamic equation of a nonlinear single-input system can be expressed as:

$$\dot{x}^n = f(x) + b(x)u \tag{56}$$

Where x represents a scalar variable and u is both the system output and control input. To choose the slide surface for the system, two conditions must be met. Firstly, the slide surface derivative \dot{s} should contain the control law u , indicating that there must be a linear relationship between them. Secondly, the system must track the desired trajectory, such that $x(t) = xd(t)$ as S approaches 0, where $xd(t)$ is the desired value.

Based on the two conditions mentioned earlier, the slide surface was chosen as follows:

$$s = x_2 + c_1 x_1 + c_2 x_4 + c_3 x_3 \tag{57}$$

Where c_1, c_2, c_3 are control parameters that can be determine by Hurwitz

After testing the first conditions to the chosen slide surface, the relationship between S and u is a linear.

$$\dot{s} = \dot{x}_1 + c_1 x_2 + c_2 \dot{x}_3 + c_3 x_4 \tag{58}$$

From eq. [50],eq. [51] and eq. [58] we got:

$$\dot{s} = f_1 + b_1 u + c_1 x_2 + c_2 (f_2 + b_2 u) + c_3 x_4 \tag{59}$$

$$\dot{s} = f_1 + f_2 c_2 + c_1 x_2 + (b_1 + c_2 b_2)u + c_3 x_4 \tag{60}$$

For the sliding mode control law design:

Defining:

$$\dot{s} = -\zeta \text{sgn}(s) \tag{61}$$

Then by obtaining the equation 62:

$$s\dot{s} = -s\zeta \text{sgn}(s) = -\zeta |s| \leq 0 \tag{62}$$

So the control designed u as:

$$u = -\frac{-1}{b_1 - c_2 b_2} [f_1 + f_2 c_2 + c_1 x_2 + c_3 x_4 + \zeta \text{sgn}(s)] \tag{63}$$

5.5 Hurwitz Stability Analysis:

Eq. (63) represents the slide mode control law for the crane system. In order to ensure stability of the system, it is necessary to find the values of the constants $c_1, c_2,$ and c_3 using the Hurwitz stability analysis method.

$$s = x_2 + c_1 x_1 + c_2 x_4 + c_3 x_3 = 0 \tag{64}$$

$$\dot{x}_1 = x_2 = -c_1 x_1 - c_2 x_4 - c_3 x_3 \tag{65}$$

$$x_4 = -\frac{-1}{l - c_2 \cos x_3} [-g \sin x_3 + \cos x_3 (c_1 (-c_1 x_1 - c_2 x_4 - c_3 x_3)) + c_3 x_4] \tag{66}$$

The system equilibrium point is:

$$(x_1, x_2, x_3, x_4) = (0, 0, 0, 0) \tag{67}$$

At the point in eq. [67], the eq. [68] is correct:

$$\sin x_3 = x_3 \cos x_3 = 1 \tag{68}$$

$$x_4 = -\frac{-g x_3 - c_1 c_3 x_3 + (-c_2 c_1 + c_3) x_4 - c_1^2 x_1}{l - c_2 + \epsilon x_3 + \epsilon x_4 + \epsilon x_1} \tag{69}$$

where ϵ is the error by linearization, then obtaining:

$$\dot{x}_1 = Ax_1 + \epsilon x_1 \tag{70}$$

Where,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ A_{21} & A_{22} & A_{23} \\ -c_3 & -c_2 & -c_1 \end{bmatrix}, \tag{71}$$

$$\epsilon = \begin{bmatrix} 0 & 0 & 0 \\ \epsilon_1 & \epsilon_2 & \epsilon_3 \\ 0 & 0 & 0 \end{bmatrix}$$

ϵ is very small value, if the matrix A as Hurwitz is designed, the stability can be obtained, and if $t \rightarrow 0$ then $(x_1, x_2, x_3, x_4) = (0, 0, 0, 0)$ so:

$$a_2 \neq L, \quad A_{21} = \frac{-g - c_1 c_3}{l - c_2}, \tag{72}$$

$$A_{22} = \frac{-c_2 c_1 + c_3}{l - c_2}, A_{21}$$

$$= \frac{c_1^2}{l - c_2}$$

$$|A - \lambda I| = 0 \tag{73}$$

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 1 & 0 \\ A_{21} & A_{22} - \lambda & A_{23} \\ c_3 & -c_2 & c_1 - \lambda \end{vmatrix} \tag{74}$$

$$\lambda^3 - (A_{22} - c_1)\lambda^2 + (-cA_{22} - A_{21} + c_2A_{23})\lambda - c_1A_{21} + c_3A_{23} = 0 \tag{75}$$

From the equation 75, 74 and 73 getting:

$$c_2 = L - \frac{g}{11} \tag{76}$$

$$c_1 = -\frac{6}{g(c_2 - L)} \tag{77}$$

$$c_2 = Lc_1 + \frac{6}{(c_2 - L)} \tag{78}$$

The following equation is resulted from Eq. (75):

$$-A_{22} + c_1 = 6 \tag{79}$$

$$-c_1A_{22} - A_{21} + c_2A_{23} = 11 \tag{80}$$

$$-c_1A_{21} - c_3A_{23} = 6 \tag{81}$$

Table 1: Design parameters for the control

Design parameters	The value
c1	0.687
c2	0.1091
c3	-4.68

Simulations were carried out using Simulink in Matlab to evaluate the proposed slide mode control laws for the Convey Crane system. The physical parameters of the system.

Table 2: Physical parameters for the Crane

physical parameters	actual value	units
length of rode L	1	meter
mass wight mC	0.5	kg
load wight ml	0.5	kg
gravitational acceleration g	9.8	m/s ²

Where set to mc = 0.5 kg, ml = 0.5 kg, l = 1 m, and g = 9.8 m/s². The designed parameters for each control law are presented in Table 1. Cosine signal was added as external noise as in 86.

$$f_n = \cos 50t \tag{82}$$

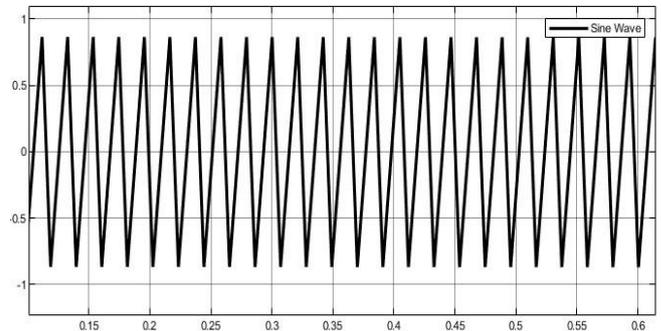


Figure 8: The external noise

The figure presented in 16 demonstrates the effectiveness of the proposed sliding mode control method in stabilizing the system, even in the presence of noise signals.

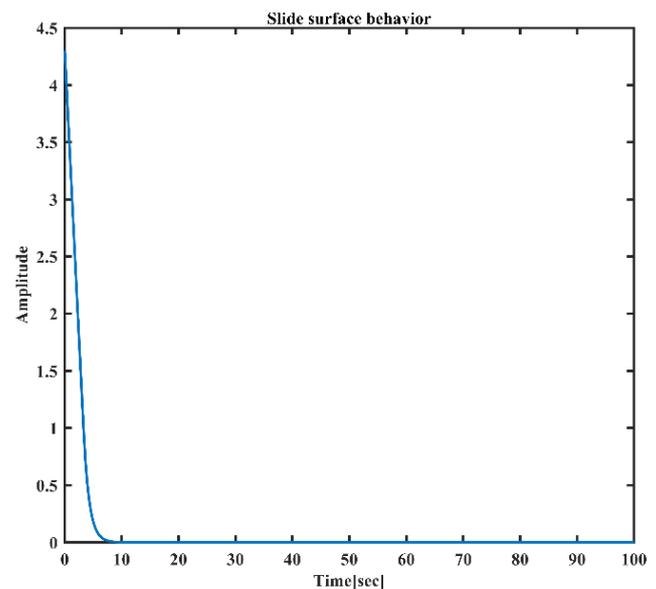


Figure 9: The slide surface behavior

As depicted in Figure 17, the position state of the system reaches the equilibrium point, and the system remains stable even in the presence of a noise signal.

As depicted in Figure 18, the velocity state reaches the equilibrium point even in the presence of noise signals, and the system remains stable thanks to the proposed sliding mode control method.

As depicted in Figure 19, the rope angle of the convey crane system reaches the equilibrium point even in the presence of a noise signal, and the system remains stable.

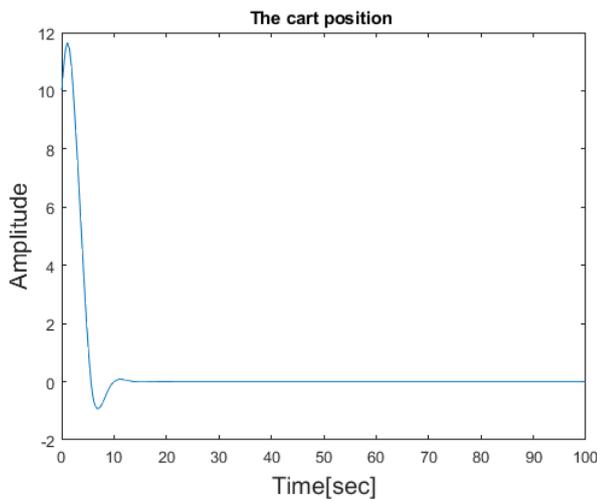


Figure 17: The cart position state of the system

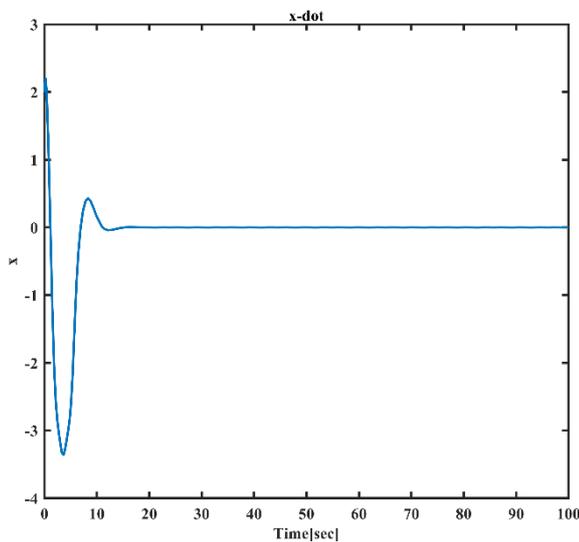


Figure 18: The cart velocity state of the system

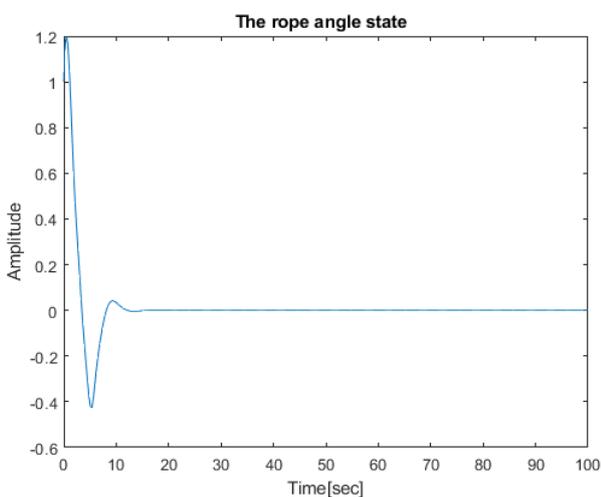


Figure 19: The rope angle state of the system

The angular velocity of the rope is successfully stabilized at the equilibrium point with the proposed sliding mode control, as demonstrated in Figure 20. Even in the presence of noise signal, the system exhibits stable behavior.

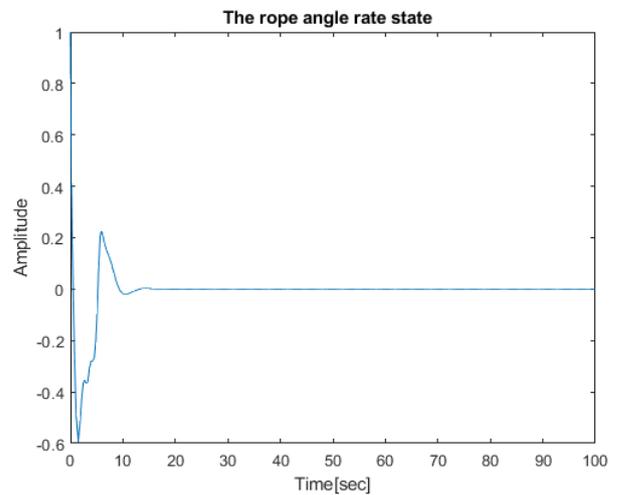


Figure 20: The rope angle rate state of the system

The results depicted in Figures 16 through 20 demonstrate the effectiveness of applying sliding mode control to the under-actuated nonlinear system in the presence of noise. Despite the noise signal, the system was able to reach the equilibrium point after the application of SMC, indicating robust stability.

VI. CONCLUSION

This paper addresses uncertainties in a convey crane system, specifically uncertain parameters and a noisy signal. The first uncertainty is addressed using state feedback control, which is designed to stabilize the system while considering the uncertainties. The state feedback control stability is tested using edge theory. The second uncertainty, which is the noisy signal, is handled using a sliding mode controller (SMC). The SMC is designed to stabilize the underactuated nonlinear convey crane system and improve its robustness against interference signals. The nonlinear differential equations of the convey crane system are presented as the basis for the control design, and simulation is performed using MATLAB Simulink. The performance of the designed controller is evaluated without and with noise signal to test its stability under different conditions. A traditional PID controller is also applied to a linear convey crane system for comparison. The study emphasizes the importance of addressing uncertainties in the design of control systems. The results show that the proposed SMC outperforms the PID controller in terms of stability and robustness. The proposed approach can be applied to similar systems with uncertainties and interference signals.

REFERENCES

- [1] Eihab M Abdel-Rahman, Ali H Nayfeh, and Ziyad N Masoud. Dynamics and control of cranes: A review. *Journal of Vibration and control*, 9(7):863–908, 2003.
- [2] Ju`rgen Ackermann, Andrew Bartlett, Dieter Kaesbauer, Wolfgang Sienel, and Reinhold Steinhauser. *Robust control: Systems with uncertain physical parameters*. Springer, 1993.
- [3] Ju`rgen Ackermann, Paul Blue, Tilman Bu`nte, L Gu`venc, Dieter Kaesbauer, Michael Kordt, Michael Muhler, and Dirk Odenthal. *Robust control: the parameter space approach*, volume 2. Springer, 2002.
- [4] Charles Aguiar, Daniel Leite, Daniel Pereira, Goran Andonovski, and Igor Skrjanc. Non-~ linear modeling and robust lmi fuzzy control of overhead crane systems. *Journal of the Franklin Institute*, 358(2):1376–1402, 2021.
- [5] Naif B Almutairi and Mohamed Zribi. Sliding mode control of a three-dimensional overhead crane. *Journal of vibration and control*, 15(11):1679–1730, 2009.
- [6] Giorgio Bartolini, Alessandro Pisano, and Elio Usai. Second-order sliding-mode control of container cranes. *Automatica*, 38(10):1783–1790, 2002.
- [7] A Benhidjeb and GL Gissingner. Fuzzy control of an overhead crane performance comparison with classic control. *Control Engineering Practice*, 3(12):1687–1696, 1995.
- [8] J. Collado, R. Lozano, and I. Fantoni. Control of convey-crane based on passivity. In *Proceedings of the 2000 American Control Conference. ACC (IEEE Cat. No.00CH36334)*, volume 2, pages 1260–1264 vol.2, 2000.
- [9] Sami Ud Din, Qudrat Khan, Fazal-Ur Rehman, and Rini Akmeliawanti. A comparative experimental study of robust sliding mode control strategies for underactuated systems. *IEEE Access*, 5:10068–10080, 2017.
- [10] Sam Chau Duong, Eiho Uezato, Hiroshi Kinjo, and Tetsuhiko Yamamoto. A hybrid evolutionary algorithm for recurrent neural network control of a three-dimensional tower crane. *Automation in Construction*, 23:55–63, 2012.
- [11] Taha Elmokadem, Mohamed Zribi, and Kamal Youcef-Toumi. Trajectory tracking sliding mode control of underactuated auvs. *Nonlinear Dynamics*, 84(2):1079–1091, 2016.
- [12] Isabelle Fantoni, Rogelio Lozano, and Rogelio Lozano. *Non-linear control for underactuated mechanical systems*. Springer Science & Business Media, 2002.
- [13] Huiru Guo, Zhi-Yong Feng, and Jinhua She. Discrete-time multivariable pid controller design with application to an overhead crane. *International Journal of Systems Science*, 51(14):2733–2745, 2020.
- [14] Hazriq Izzuan Jaafar, Z Mohamed, Amar Faiz Zainal Abidin, and Z Ab Ghani. Pso-tuned pid controller for a nonlinear gantry crane system. In *2012 IEEE International Conference on Control System, Computing and Engineering*, pages 515–519. IEEE, 2012.
- [15] Gyoung-Hahn Kim and Keum-Shik Hong. Adaptive sliding-mode control of an offshore container crane with unknown disturbances. *IEEE/ASME Transactions on Mechatronics*, 24(6):2850–2861, 2019.
- [16] Ho-Hoon Lee. Modeling and control of a three-dimensional overhead crane. 1998.
- [17] Lun-Hui Lee, Pei-Hsiang Huang, Yu-Cheng Shih, Tung-Chien Chiang, and Cheng-Yuan Chang. Parallel neural network combined with sliding mode control in overhead crane control system. *Journal of Vibration and Control*, 20(5):749–760, 2014.
- [18] Javier Moreno-Valenzuela and Carlos Aguilar-Avelar. *Motion control of underactuated mechanical systems*, volume 1. Springer, 2018.
- [19] Quang Hieu Ngo, Ngo Phong Nguyen, Chi Ngon Nguyen, Thanh Hung Tran, and Quang Phuc Ha. Fuzzy sliding mode control of an offshore container crane. *Ocean Engineering*, 140:125–134, 2017.
- [20] Reza Olfati-Saber. *Nonlinear control of underactuated mechanical systems with application to robotics and aerospace vehicles*. PhD thesis, Massachusetts Institute of Technology, 2001.
- [21] Pathan Shabnam, AK Priyanka, T Vijay Muni, and S Rajasekhar. Pid controller based grid connected wind turbine energy system for power quality improvement. *Journal of Critical Reviews*, 7(7):31–35, 2020.
- [22] Le Anh Tuan and Soon-Geul Lee. Sliding mode controls of double-pendulum crane systems. *Journal of Mechanical Science and Technology*, 27(6):1863–1873, 2013.
- [23] Le Anh Tuan, Sang-Chan Moon, Won Gu Lee, and Soon-Geul Lee. Adaptive sliding mode control of overhead cranes with varying cable length. *Journal of Mechanical Science and Technology*, 27(3):885–893, 2013.
- [24] Xianqing Wu, Kexin Xu, Meizhen Lei, and Xiongxiang He. Disturbance-compensation based continuous sliding mode control for overhead cranes with disturbances. *IEEE Transactions on Automation Science and Engineering*, 17(4):2182–2189, 2020.
- [25] Jian-Xin Xu, Zhao-Qin Guo, and Tong Heng Lee. Design and implementation of integral sliding-mode control on an underactuated two-wheeled mobile robot.

- IEEE Transactions on industrial electronics, 61(7):3671–3681, 2013.
- [26] Jung Hua Yang and Kuang Shine Yang. Adaptive coupling control for overhead crane systems. *Mechatronics*, 17(2-3):143–152, 2007.
- [27] Wen Yu, Xiaou Li, and Francisco Panuncio. Stable neural pid anti-swing control for an overhead crane. *Intelligent Automation & Soft Computing*, 20(2):145–158, 2014.
- [28] Menghua Zhang, Xin Ma, Rui Song, Xuewen Rong, Guohui Tian, Xincheng Tian, and Yibin Li. Adaptive proportional-derivative sliding mode control law with improved transient performance for underactuated overhead crane systems. *IEEE/CAA Journal of Automatica Sinica*, 5(3):683–690, 2018.
- [29] Network lifetime through an integrated model for clustering and routing in wireless sensor networks,” *Comput. Netw.*, vol. 55, no. 13, pp. 2803–2820, Sep. 2011.

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