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Dissemination of Suzuki Contraction Theorem in a Metric Space

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Abstract **- A novel and robust extension of the traditional Banach contraction theorem is the Suzuki contraction theorem [47]. One of the most prominent statements in the field of metric fixed-point is this theorem. A comprehensive overview of the Suzuki contraction theorem's developments over the past 15 years is covered in this study. This paper offers an explanation of the literature that advances the Suzuki contraction theorem by improving, extending, and generalizing it to metric spaces.**

2020 Mathematics Subject Classifications: 47H10; 54H25

Keywords: Contractions; metric completeness; coincidence point; fixed point; common fixed point; coupled fixed points; IT-commuting.

I. Introduction

For the sake of conciseness, we adhere to the following functions throughout the paper.

(i) A function μ that is non-increasing from [0, 1) to $\left(\frac{1}{2},\, 1\right]$ is defined as

$$\mu(a) = \begin{cases} 1 \text{ if } 0 \le a \le \frac{1}{2}(\sqrt{5}-1), \\ \frac{1-a}{a^2} \text{ if } \frac{1}{2}(\sqrt{5}-1) \le a \le \frac{1}{\sqrt{2}}, \\ \frac{1}{1+a} \text{ if } \frac{1}{\sqrt{2}} \le a < 1. \end{cases}$$

(ii) A function ξ that is non-increasing from [0, 1) to $\left(\frac{1}{2},\, 1\right]$ is defined as

$$\xi(a) = \begin{cases} 1 & \text{if } 0 \le a \le \frac{1}{2} \\ \frac{1}{1+a} & \text{if } \frac{1}{2} \le a < 1. \end{cases}$$

(iii) A function ς that is non-increasing from [0, 1) to $\left(\frac{1}{2},\, 1\right]$ is defined as

$$\varsigma(a) = \begin{cases} 1 & \text{if } 0 \le a \le \frac{1}{\sqrt{2}} \\ \frac{1}{1+a} & \text{if } \frac{1}{\sqrt{2}} \le a < 1. \end{cases}$$

(iv) A function τ that is non-increasing from [0, 1) to (0, 1] is defined as

$$\tau(a) = \begin{cases} 1 & \text{if } 0 \le a < \frac{\sqrt{5}-1}{2} \\ 1-a & \text{if } \frac{\sqrt{5}-1}{2} \le a < 1. \end{cases}$$

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(v) A function χ that is non-increasing from [0, 1) to (0, 1] isdefined as

$$\chi(a) = \begin{cases} 1 & \text{if } 0 \leq a < \frac{1}{2} \\ 1 - a & \text{if } \frac{1}{2} \leq a < 1. \end{cases}$$

(vi) A function Ω that is strictly decreasing from [0, 1) to $\left(\frac{1}{2}, 1\right]$ is defined as

$$\Omega(a) = \frac{1}{1+a}$$

Symbols utilized:

$\mathcal{L}$, $\mathcal{M}$	Subsets of $\mathcal{R}$ which are Non-empty
$(\mathcal{L}, m_s)$	Metric space
$CL(\mathcal{L})$	Collection of subsets of $\mathcal{L}$ which are closed and non-empty
$CB(\mathcal{L})$	Collection of subsets of $\mathcal{L}$ which are closed, bounded and non-empty
$\exists$	There exists
$\forall$	For all
$\in$	Belongs to

Abbreviations and Acronyms:

FP	Fixed Point
CdFP	Coupled Fixed Point
BCP	Banach Contraction Principle
MS	Metric space
CMS	Complete Metric Space
CoMS	Compact Metric Space
CP	Coincidence Point
CdCP	Coupled Coincidence Point
CFP	Common Fixed Point
CdCFP	Coupled Common Fixed Point
CS	Complete subspace
ITC	IT-commuting

There arises a situation where the solutions of the system of equations cannot be directly traversed frequently. As a result, a reasonable query that "Is there a solution for the given system of equations?"

After being addressed, the question "How many different solutions exist for the given system of equations?" is posed.

This motivates them to do a more thorough advanced research study, which is an essential precondition for researchers to delve thoroughly into the idea of fixed points. By employing suitable methods, the system of equations can be reformulated as the computation of fixed points or common fixed points of self-mapping(s) specified over a suitable space $\mathcal{L}$.

Analytically, fixed point of a self-mapping $\mathcal{O}$ defined on an appropriate set or space signifies the intersection of the curve $e = \mathcal{O}d$ with the line $e = d$. In an alternative approach, a solution to the equation $\mathcal{O}d = d$ can be used to define a FP of the mapping $\mathcal{O}$ i.e. remains invariant under the mapping $\mathcal{O}$. There are numerous applications for the theory of fixed points in many different fields. There are many formulations in fixed point theory by various authors; one of them is BCP which was introduced by S. Banach [6] in 1922 as follows:

Theorem 1.1 [6]. Let $(\mathcal{L}, m_s)$ be a CMS and let $:\mathcal{L} \to \mathcal{L}$ be such that for each $d, e \in \mathcal{L}$,

$$m_s(\mathcal{O}d, \mathcal{O}e) \leq a m_s(d, e), \text{ where } a \in [0, 1).$$

Then $\mathcal{O}$ has a unique FP r in $\mathcal{L}$. Moreover, for an arbitrary point $d_0 \in \mathcal{L}$, $\mathcal{O}^n d_0 \to r$.

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It's interesting to note that, in a contraction mapping, the distance between each pair of images contracts (shrines) relative to the element's distance.

BCP has become a standard tool in nonlinear analysis due to its ease of use and simplicity. It's one of the most well-known fixed point theory concepts, and many mathematicians, scientists, and economists have expanded, generalized, and united it for both single-valued and multivalued maps under various contractive conditions in different spaces. (see, for instance, [9], [10-16], [18], [19], [20-25], [26-27], [30-31], [37-38] and [40]-[50]).

To obtain a basic comparison of different contractive conditions for one and two mappings, look up Rhoades [39].

A selfmap $\mathcal{O}$ of $\mathcal{L}$ is *strictly contractive* if

$$m_s(\mathcal{O}d, \mathcal{O}e) < m_s(d, e) \qquad (1.1)$$

for all $d, e \in \mathcal{L}$, $d \neq e$.

It is not necessary for a map $\mathcal{O}$ satisfying (1.1) to have a fixed point on aCMS. In fact, Edelstein [16] demonstrated that $\mathcal{O}$ satisfying (1.1) has a unique FP on a compact metric. Indeed, Edelstain's theorem can be restated under the following invention:

Theorem 1.2 [16] Let $(\mathcal{L}, m_s)$ be a CoMS and $\mathcal{O}: \mathcal{L} \to \mathcal{L}$ a strict contractive map. Then $\mathcal{O}$ admit a unique FP in $\mathcal{L}$.

We observe that iterations of $\mathcal{O}$ that satisfy (1.1) do not necessarily converge, and if they do, it will be to a FP. Furthermore, $\mathcal{O}$ satisfying (1.1) does not necessarily have to have a FP in a CMS. Thus, the need for compactness is essential. For example, let $\mathcal{L}$ = [1, ∞) and $\mathcal{O}:\mathcal{L} \to \mathcal{L}$ with $\mathcal{O}d = d + 1/d$. Then $|\mathcal{O}d - \mathcal{O}e| < |d - e|$ hold for all distinct d, e in $\mathcal{L}$. Evidently, $\mathcal{K}$ has no FP.

Given that BCP generates fixed points for continuous self-mappings, a general question is, "Is it possible to produce fixed points for mappings subjected to contraction without being continuous on the entire domain?" Certain affirmative replications can be found as we read through the literature.

By adopting an entirely different condition, Kannan [21] proved a FP theorem for operators, which need not be contractions. Actually, the FP theorem for a discontinuous map was first put forth by Kannan. The FP theorem that follows, also known as the Kannan contraction theorem (KCP), was in fact demonstrated by Kannan [21].

Theorem 1.3 Let $\mathcal{L}$ be a CMS and let $\mathcal{O}:\mathcal{L}\to\mathcal{L}$. Assume there exists $a \in [0, 1/2)$ such that for each $d, e \in \mathcal{L}$,

$$m_s(\mathcal{O}d, \mathcal{O}e) \leq a m_s(d, \mathcal{O}d) + a m_s(e, \mathcal{O}e).$$

Then $\mathcal{O}$ has a unique FP in $\mathcal{L}$.

Kannan's theorem motivated numerous extensions and generalizations of the BCP and KCP.

Another intriguing result, known as the Meir-Keeler contraction, was compiled in 1969 under stringent contraction conditions by Meir and Keeler [29] (see also, [3], [33-36]) and is provided below:

Theorem 1.4 A self-map $\mathcal{O}$ of a CMS $(\mathcal{L}, m_s)$ fulfilling the requirement:

for a given $\varepsilon > 0, \exists$ a $\delta > 0$ such that for every $d, e \in L$,

$$\varepsilon \leq m_s(d, e) < \varepsilon + \delta \text{ implies } m_s(\mathcal{O}d, \mathcal{O}e) < \varepsilon,$$

has a unique FP.

In 1941, Kakutani began researching FP problems for multivalued maps in finite dimensional spaces. Bohnenblust and Karlin extended it to an infinite dimensional Banach space in 1950, and Ky Fan extended it to a locally convex space in 1952 (see [49]). Nadler Jr. [31] (see also [32]) in 1969, introduced the concept of multivalued contraction mappings motivated by the BCT.

Let $(\mathcal{L}, m_s)$ be a MS. Consistent with Nadler, Jr. [32, p. 620], take Hausdorff metric, as

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$$\mathcal{H}(A, B) = \max\left\{\sup_{d\in A} m_s(d, B),\ \sup_{e\in B} m_s(e, A)\right\},$$

For $A, B \in CB(\mathcal{L})$ and $m_s(d, B) = \inf_{e\in B} m_s(d, e)$.

Nadler [31] provided the following result, which is the multivalued version of Theorem 1.1 and can be referred to as the Nadler FP theorem.

Theorem 1.5 Let $\mathcal{L}$ be a CMS and let $\mathcal{O}: \mathcal{L}\to CB(\mathcal{L})$. Assume $\exists\, a\in [0, 1)$ such that for each $d, e\in\mathcal{L}$

$$\mathcal{H}(\mathcal{O}d, \mathcal{O}e) \leq a m_s(d, e).$$

Then there exists $z\in\mathcal{L}$ such that $z\in\mathcal{O}z$.

Later on Nadler Jr. replaced the condition "$\mathcal{O}: \mathcal{L}\to CB(\mathcal{L})$" in Theorem 1.6 by "$\mathcal{O}: \mathcal{L}\to CL(\mathcal{L})$" (see [32] for details).*Nadler multivalued contraction theorem* received a lot of attention in applicable mathematics, especially in solving inclusions and multivalued differential & functional equations, and was extended and generalized on various settings.

The result was expanded to multivalued contractions by Ćirič [8]. A map $\mathcal{O}: \mathcal{L}\to CL(\mathcal{L})$ is generalized contraction iff there exists $0 \leq a < 1$ such that, for every $d, e\in\mathcal{L}$,

$$\mathcal{H}(\mathcal{O}d, \mathcal{O}e) \leq a\max\{m_s(e, \mathcal{O}e), m_s(d, e), m_s(d, \mathcal{O}d), [m_s(d, \mathcal{O}e) + m_s(e, \mathcal{O}d)]/2\}.$$

Till now, contractions consisting of the single self-mappings have been presented. In consideration of the readers' interest, we will now present some significant findings, which include two self-mappings.CFP problems make up an important class in the theory of FP. For the two "commuting mappings," Jungck [19] developed a noteworthy result in this regard.

K. Goebel (1968) and G.Jungck [19] (1976) examined the following condition for a pair of maps $\mathcal{O}, \mathcal{F}: \mathcal{M}\to\mathcal{L}$, when $\mathcal{M} = \mathcal{L}$, see [22, 29]):

$$m_s(\mathcal{O}d, \mathcal{O}e) \leq m_s(\mathcal{F}d, \mathcal{F}e) \qquad (1.2)$$

for all $d, e \in M, a\in [0, 1)$, where $\mathcal{M}$ is arbitrary nonempty set and $\mathcal{O}(\mathcal{M})$ is contained in

$$\mathcal{F}(\mathcal{M}).$$

While some mathematicians were aware of the condition (1.2) prior to 1976 (refer to Kulshrestha [28]), Jungck [19] is recognized for providing a constructive demonstration of the existence of a CFP of commuting maps $\mathcal{K}$ and $\mathcal{F}$ that satisfies (1.2) along with a few more conditions. Truly, he demonstrated that:

Theorem 1.6 Let $(\mathcal{L}, m_s)$ be a CMS and $\mathcal{O}, \mathcal{F}: \mathcal{L}\to\mathcal{L}$ with $f(\mathcal{L})$ is superset of $\mathcal{O}(\mathcal{L})$ and (1.2) for all $d, e \in L$. If $\mathcal{O}$ and $\mathcal{F}$ are commuting on $\mathcal{L}$ and $\mathcal{F}$ is continuous, then $\mathcal{O}$ and $\mathcal{F}$ have a unique CFP.

Singh has, nevertheless, provided an enhanced version of Jungck's above theorem, which can be used in situations where a contractive method is being used to solve a pair of equations. For two maps, the same is commonly referred to as the Jungck-Singh contraction theorem.

The aforementioned theorems, the Kannan contraction theorem results, and condition (1.2) have all contributed to the enormous expansion of FP theorems and their various applications. Nevertheless, the contractive or contractive conditions must hold for every point $\mathfrak{b}, c$ in the domain in order for any of these FP theorems to apply. Thus, one should naturally anticipate the day when this condition is significantly loosened without compromising the theorem's conclusions. This laid the groundwork for fixed point theorems of the Suzuki type.

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II. Suzuki-Type Fixed Point Theorems for Single-Valued Contractions

This section presents the Suzuki contraction theorem and a few of the developments made to it during the last two and half decades, including extensions and generalizations (see for instance, [1-2], [4-5], [7], [10-12], [14], [17], [20], [22-27], [30], [37-38] and [40-49]).

The next theorem, which is referred to as the Suzuki contraction theorem, was obtained by Suzuki [47]. It characterizes the metric completeness and is a new kind of generalization of BCP.

Theorem 2.1 [63]. Let $\mathcal{L}$ be a CMS and let $\mathcal{O}$: $\mathcal{L} \to \mathcal{L}$. Assume $\exists\, a \in [0, 1)$ so that for all $\mathcal{b}$, $c \in \mathcal{L}$,

$$\mu(a)\, m_s(d, \mathcal{O}d) \leq m_s(d, e) \text{ implies } m_s(\mathcal{O}d, \mathcal{O}e) \leq a m_s(d, e).$$

Then $\mathcal{O}$ has a unique FP. Moreover, $\lim_{x\to\infty} \mathcal{O}^n d = r$ for any d in $\mathcal{L}$.

The next illustration demonstrates how Theorem 2.1 is a broader elaboration of Theorem 1.1.

Example 2.1 [47].Consider metric space

$$\mathcal{L} = \{(1,1), (4,1), (1,4), (4,5), (5,4)\},$$

with the metric m_s as $m_s[(d_1, d_2), (e_1, e_2)] = |d_1 - e_1| + |d_2 - e_2|$.

Define a map $\mathcal{O}$ on $\mathcal{L}$ by

$$\mathcal{O}(d_1, d_2) = \begin{cases} (1,\ d_2) & \text{if } d_1 > d_2 \\ (d_1, 1) & \text{if } d_1 \leq d_2. \end{cases}$$

Then all of Theorem 2.1's hypotheses are satisfied by $\mathcal{O}$, but not by Theorem 1.1.

Proof: We first note that

$$m_s(\mathcal{O}d, \mathcal{O}e) \leq \frac{4}{5} m_s(d,\ e) \text{ if } (d, e) \neq ((4, 5), (5, 4)) \text{ and } (e, d) \neq ((4, 5), (5, 4)).$$

Now at (1, 1)

$\mathcal{O}(1, 1) = (1, 1)$. Note that the FP (1, 1) is unique.

Thus $\mathcal{O}$ satisfies the assumptions in Theorem 2.1.

But when $d = (4, 5)$, $e = (5, 4)$

$$m_s(\mathcal{O}d, \mathcal{O}e) = 6, m_s(d, e) = 2.$$

Thus $\mathcal{O}$ does not satisfy the assumptions in Theorem 1.1 for any $a \in [0, 1)$.

Remark: It is not necessary for the Suzuki contraction to be continuous.

Researchers have made numerous significant additions and generalizations over the years. As a result, there is a vast ocean of research on this topic, a few drops of which are included in the current paper.

The Kannan version of the Suzuki contraction theorem was demonstrated in the following way by Kikkawa and Suzuki [25] in 2008.

Theorem 2.3 Let $\mathcal{L}$ be a CMS and let $\mathcal{O}$: $\mathcal{L} \to \mathcal{L}$. Let $\alpha \in \left[0,\ \frac{1}{2}\right)$ and put $a = \frac{\alpha}{(1-\alpha)} \in [0, 1)$. Assume $\exists\, a \in [0, 1)$ in a way that for every d, $c \in \mathcal{L}$,

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$$\varsigma(a)\, m_s(d, \mathcal{O}d) \leq m_s(d, e)$$

implies

$$m_s(\mathcal{O}d, \mathcal{O}e) \leq \alpha m_s(d, \mathcal{O}d) + \alpha m_s(e, \mathcal{O}e).$$

Then $\mathcal{O}$ has a unique FP.

Remark: Since $\mu(a) \leq \varsigma(a)$ for every a, so from a Kikkawa-Suzuki perspective, we can conclude that the Kannan version is stronger. Moreover, they [25] have noted that the graphs of μ and ς are quite similar.

The work of Kikkawa and Suzuki [25] was generalized in the following way by Mot and Petrusel [30] with their new contraction type, the (α, β, γ) Mot-Petrusel- Suzuki contraction:

Theorem 2.4 Let $\mathcal{L}$ be a CMS and let $\mathcal{O}$: $\mathcal{L} \rightarrow \mathcal{L}$. If $\exists \alpha, \beta, \gamma \in \mathcal{R}_+$ with $\alpha + \beta + \gamma \in [0, 1)$in a way that for every $d, e \in \mathcal{L}$,

$$\frac{1-\beta-\gamma}{1+\alpha} m_s(d, \mathcal{O}d) \leq m_s(d, e)$$

implies

$$m_s(\mathcal{O}d, \mathcal{O}e) \leq \alpha m_s(d, e) + \beta m_s(d, \mathcal{O}d) + \gamma m_s(e, \mathcal{O}e).$$

Then $\mathcal{O}$ admit a unique FP.

To try to make Theorem1.6 more expansive, Kikkawa and Suzuki [25] provided the following result for a novel kind of contraction known as generalized Kikkawa-Suzuki-Kannan-type contraction.

Theorem 2.5 Let :$\mathcal{L} \rightarrow \mathcal{L}$ with $\mathcal{L}$ a CMS. Assume $\exists\, a \in [0, 1)$ such that for each $d, e \in \mathcal{L}$,

$$\mu(a)\, m_s(d, \mathcal{O}d) \leq m_s(d, e)$$

implies

$$m_s(\mathcal{O}d, \mathcal{O}e) \leq a \max\{m_s(d, \mathcal{O}d), m_s(e, \mathcal{O}e)\}.$$

Then $\mathcal{O}$ has a unique FP.

By extending Theorem 2.5 by Kikkawa and Suzuki [25], Dhompongsa and Yingtaweesittikul [12] produced a more general class of contraction mappings.

Theorem 2.6 Consider $\mathcal{L}$ to be a CMS and :$\mathcal{L} \rightarrow \mathcal{L}$. Assume $\exists\, a \in [0, 1)$ such that for each $c, d \in \mathcal{L}$,

$$\mu(a)\, m_s(d, \mathcal{O}d) \leq m_s(d, e)$$

implies

$$m_s(\mathcal{O}d, \mathcal{O}e) \leq a \max\{m_s(d, e), m_s(d, \mathcal{O}d), m_s(e, \mathcal{O}e)\}.$$

Then $\mathcal{O}$ has a unique FP.

Kikkawa and Suzuki [26] provided another significant Suzuki type FP result under the topic of commuting maps in the subsequent manner.

Theorem 2.7 Let $\mathcal{L}$ be a CMS,$\mathcal{O}$: $\mathcal{L} \rightarrow \mathcal{L}$, and a continuous map $\mathcal{F}$: $\mathcal{L} \rightarrow \mathcal{L}$ with $\mathcal{F}(\mathcal{L})$ is superset of $\mathcal{O}(\mathcal{L})$, and on $\mathcal{L}$, both $\mathcal{F}$ and $\mathcal{O}$ are commuting. Assume $\exists\, a \in [0, 1)$ such that for each $d, e \in \mathcal{L}$,

$$\mu(a)\, m_s(fd, \mathcal{O}d) \leq m_s(\mathcal{F}d, \mathcal{F}e) \text{ implies } m_s(\mathcal{O}d, \mathcal{O}e) \leq a m_s(\mathcal{F}d, \mathcal{F}e).$$

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Then ∃a unique CFP of $\mathcal{F}$ and $\mathcal{O}$.

Another result was provided for two contractive non-self-maps in 2011 by Singh and Mishra [42]. The following was acquired by them:

Theorem 2.8 Let $\mathcal{L}$ be a MS and $\mathcal{M}$ an arbitrary non-empty set. Consider $\mathcal{F}, \mathcal{O}: \mathcal{M} \to \mathcal{L}$ with

$\mathcal{F}(\mathcal{M})$ is superset of $\mathcal{K}(\mathcal{M})$, and $\mathcal{O}(\mathcal{M})$ or $\mathcal{F}(\mathcal{M})$ is CS of $\mathcal{L}$. Assume $\exists \alpha, \beta, \gamma \in [0, 1)$ with $\alpha + 2\beta + 2\gamma < 1$, and for every $d, c \in \mathcal{L}$,

$$\frac{1-\beta-\gamma}{1+\alpha} m_s(\mathcal{F}d, \mathcal{O}d) \le m_s(\mathcal{F}d, \mathcal{F}e)$$

implies

$$m_s(\mathcal{O}d, \mathcal{O}e) \le a m_s(\mathcal{F}d, \mathcal{F}e) + \beta[m_s(\mathcal{F}d, \mathcal{O}d) + m_s(\mathcal{F}e, \mathcal{O}e)] + \gamma[m_s(\mathcal{F}d, \mathcal{O}e) + m_s(\mathcal{F}e, \mathcal{O}d)].$$

Then $\mathcal{F}$ and $\mathcal{O}$ have a CP, that is, there exists $r \in \mathcal{M}$ such that $fr = \mathcal{O}r$.

Further, if $\mathcal{M} = \mathcal{L}$, then $\mathcal{F}$ and $\mathcal{O}$ have a CFP provided that $\mathcal{F}$ and $\mathcal{O}$ are ITC (just) at r and $\mathcal{O}r$ is FP of $\mathcal{O}$.

By extending the previous result to the class of Ćirić's generalized contractions, O. Popescu [37] obtained the following for two commuting maps in 2009.

Theorem 2.9 Let $\mathcal{L}$ be a CMS, $\mathcal{O}: \mathcal{L} \to \mathcal{L}$ and a continuous map $\mathcal{F}: \mathcal{L} \to \mathcal{L}$ with $\mathcal{F}(\mathcal{L})$ is superset of $(\mathcal{L})$, and on $\mathcal{L}$, both $\mathcal{F}$ and $\mathcal{O}$ are commuting. Assume $\exists a \in [0, 1)$ such that for each $b, c \in \mathcal{L}$,

$$\mu(a)\, m_s(\mathcal{F}d, \mathcal{O}d) \le m_s(\mathcal{F}d, \mathcal{F}e)$$

implies

$$m_s(\mathcal{O}d, \mathcal{O}e) \le a \max\left\{m_s(\mathcal{F}d, Fe), m_s(\mathcal{F}d, \mathcal{O}d), m_s(\mathcal{F}e, \mathcal{O}e), \frac{m_s(\mathcal{F}d, \mathcal{O}e) + m_s(\mathcal{F}e, \mathcal{O}d)}{2}\right\}.$$

Then ∃a unique CFP of $\mathcal{F}$ and $\mathcal{O}$.

The subsequent outcome for a pair of maps is innovatively presented by Singh et al. [40].

Theorem 2.10 Let $\mathcal{L}$ be a CMS and let $\mathcal{J}, \mathcal{O}: \mathcal{L} \to \mathcal{L}$. If $\exists \alpha, \beta, \gamma, \delta, \theta \in [0, 1)$ so that $\alpha + \beta + \gamma + \delta + \theta < 1$ and, with $\beta = \gamma$ or $\delta = \theta$ so that

$$\frac{1-\beta-\gamma-\delta-\theta}{1+\alpha+\delta+\theta} \min\{m_s(d, \mathcal{J}d), m_s(e, \mathcal{O}e)\} \le m_s(d, e)$$

implies

$$m_s(\mathcal{J}d, \mathcal{O}e) \le \alpha m_s(d, e) + \beta m_s(d, \mathcal{J}d) + \gamma m_s(e, \mathcal{O}e) + \delta m_s(d, \mathcal{J}e) + \theta m_s(e, \mathcal{O}d),$$

for all $d, e \in \mathcal{L}$.

Then ∃a unique CFP of $\mathcal{J}$ and $\mathcal{O}$.

III. Fixed Point Results for Multi-Valued Contraction

Notably, the Suzuki contraction theorem, along with its extensions and generalizations examined in the preceding section, results in fixed points for self-mappings. Therefore, the question "whether is it possible to produce fixed points for non-self-mapping?" is asked in a general way. Researchers have always been more interested in studying mappings whose range is CL($\mathcal{L}$) or CB($\mathcal{L}$), which is what we study in this section, than in working on self-mapping. By Suzuki and Kikkawa [49], the first step in this direction was proposed.

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This outcome, which is essentially a generalization of Theorem 2.1, was demonstrated for set-valued maps by Suzuki and Kikkawa [49].

Theorem 3.1 Let $\mathcal{O}$: $\mathcal{L} \to CB(\mathcal{L})$ where $(\mathcal{L}, m_s)$ be a CMS. Assume $\exists\, a \in [0, 1)$ such that for each d, $c \in \mathcal{L}$

$$\xi(a)\, m_s(d, \mathcal{O}d) \leq m_s(d, e) \text{ implies } \mathcal{H}(\mathcal{O}d, \mathcal{O}e) \leq a m_s(d, e).$$

Then there exists $\imath \in \mathcal{L}$ such that $\imath \in \mathcal{O}\imath$.

Assuming the following, Kikkawa and Suzuki [27] proved a slight modification of above theorem

Theorem 3.2 Let $\mathcal{L}$ be a CMS and let $\mathcal{O}$: $\mathcal{L} \to CB(\mathcal{L})$. Assume $\exists\, a \in [0, 1)$ such that for each d, $e \in \mathcal{L}$

$$\xi(a)\, m_s(d, \mathcal{O}d) \leq m_s(d, e) \text{ implies } \mathcal{H}(\mathcal{O}d, \mathcal{O}e) \leq a m_s(d, e).$$

Then $\exists\, \imath \in \mathcal{L}$ such that $\imath \in \mathcal{O}\imath$.

Singh and Mishra [43] further generalized Suzuki contraction theorem for set-valued maps in the subsequent manner.

Theorem 3.3 Let $\mathcal{L}$ be a CMS and $:\mathcal{L} \to CB(\mathcal{L})$. Assume $\exists\, a \in [0, 1)$ such that for each d, $e \in \mathcal{L}$

$$\Omega(a)\, m_s(d, \mathcal{O}d) \leq m_s(d, e) \text{ implies } \mathcal{H}(\mathcal{O}b, \mathcal{O}e) \leq a m_s(d, e).$$

Then there exists $\imath \in \mathcal{L}$ such that $\imath \in \mathcal{O}\imath$.

Furthermore, the theorem above is extended in the following way by Singh and Mishra [37] using a new function Ω.

Theorem 3.4 Let $\mathcal{L}$ be a CMS and let $:\mathcal{L} \to \mathrm{CL}(\mathcal{L})$. Assume $\exists\, a \in [0, 1)$ such that for each d, $c \in \mathcal{L}$,

$$\Omega(a) m_s(d, \mathcal{O}d) \leq m_s(d, e) \text{ implies } \mathcal{H}(\mathcal{O}d, \mathcal{O}e) \leq a m_s(d, e).$$

Then $\mathcal{O}$ has a FP.

Using a more generalized contraction, Moţ and Petruşel [30] generalized Theorem 3.1 and used it to demonstrate the subsequent intriguing outcome.

Theorem 3.5 Let $\mathcal{L}$ be a CMS and $:\mathcal{L} \to CL(\mathcal{L})$. If $\exists \alpha, \beta, \gamma \in [0, 1)$ with $\alpha + \beta + \gamma \in [0, 1)$ andfor each d, $c \in \mathcal{L}$,

$$\frac{1-\beta-\gamma}{1+\alpha} m_s(d, \mathcal{O}d) \leq m_s(d, e) \text{ implies } \mathcal{H}(\mathcal{O}d, \mathcal{O}e) \leq \alpha\, m_s(d, e) + \beta m_s(d, \mathcal{O}d) + \gamma m_s(e, \mathcal{O}e).$$

Then $\mathcal{O}$ has a FP.

The following is how Damjanović and Đorić [11] used a novel kind of mapping condition to establish a new FP result.

Theorem 3.6 Let $\mathcal{L}$ be a CMS and let $:\mathcal{L} \to CL(\mathcal{L})$.Assume $\exists \in [0, 1)$ such that for each d, $c \in \mathcal{L}$,

$$\tau(a)\, m_s(d, \mathcal{O}d) \leq m_s(d, e) \text{ implies } \mathcal{H}(\mathcal{O}d, \mathcal{O}e) \leq a \max\{m_s(d, \mathcal{O}d),\, m_s(e, \mathcal{O}e)\}.$$

Then $\mathcal{O}$ has a FP.

Under a small additional assumption, Dhompongsa and Yingtaweesittikul [12] presented the result for multivalued maps as under.

Theorem 3.7 Let $(\mathcal{L}, m_s)$ be a MS. After that, the following are identical:

(i) For each $a \in [0, 1)$, let $\mathcal{O}$: $\mathcal{L} \to CL(\mathcal{L})$ be such that for every $d, e \in \mathcal{L}$,

$$\mu(a)\, m_s(d, \mathcal{O}d) \leq m_s(d, e) \text{ implies}$$

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$$\mathcal{H}(\mathcal{O}d, \mathcal{O}e) \leq a\max\{m_s(d, e), m_s(d, \mathcal{O}d), m_s(e, \mathcal{O}e)\}.$$

(ii) $\mathcal{L}$ is complete.

If the function $\mathcal{L} \to m_s(d, \mathcal{O}d)$ is lower semi-continuous, then $\mathcal{O}$ has a fixed point.

A significant finding was developed by Đorić and Lazović [14] in 2011 by taking into consideration the following contraction, one of the many generalizations of the Suzuki contraction theorem.

Theorem 3.8 Let $\mathcal{L}$ be a CMS and let $:\mathcal{L} \to CB(\mathcal{L})$. Assume $\exists\, a \in [0, 1)$ so that for each $d, c \in \mathcal{L}$,

$$\chi(a)\, m_s(d, \mathcal{O}d) \leq m_s(d, e)$$

implies

$$\mathcal{H}(\mathcal{O}d, \mathcal{O}e) \leq a \max\left\{m_s(d, \mathcal{O}d),\ m_s(d, e),\ m_s(e, \mathcal{O}e),\ \frac{m_s(d, \mathcal{O}e) + m_s(e, \mathcal{O}d)}{2}\right\}.$$

Then, $\mathcal{O}$ has a FP.

Additionally, Singh and Mishra [43] generalized Singh and Mishra's [37] Theorem 3.3 to obtain the following coincidence theorem for a pair of single-valued and multivalued maps on any nonempty set using the idea of ITC maps.

Theorem 3.9 Let $\mathcal{L}$ be a MS and $\mathcal{M}$ an arbitrary non-empty set. Let $\mathcal{O}: \mathcal{M} \to CL(\mathcal{L})$ and $\mathcal{F}: \mathcal{M} \to \mathcal{L}$ with $\mathcal{F}(\mathcal{M})$ is superset of $\mathcal{O}(\mathcal{M})$, and $\mathcal{O}(\mathcal{M})$ or $\mathcal{F}(\mathcal{M})$ is CS of $\mathcal{L}$. Assume $\exists\, a \in [0, 1)$ so that for all $d, e \in \mathcal{L}$,

$$\Omega(a)\, m_s(d, \mathcal{O}d) \leq m_s(\mathcal{F}d, \mathcal{F}e) \text{ implies } \mathcal{H}(\mathcal{O}d, \mathcal{O}e) \leq a m_s(\mathcal{F}d, \mathcal{F}e).$$

Then $\mathcal{F}$ and $\mathcal{O}$ have a CP, that is, there exists $r \in \mathcal{M}$ such that $fr = \mathcal{O}r$.

Further, if $\mathcal{M} = \mathcal{L}$, then $\mathcal{F}$ and $\mathcal{O}$ have a CFP provided that $\mathcal{F}$ and $\mathcal{O}$ are ITC (just) at r and $\mathcal{F}r$ is FP of $\mathcal{F}$.

From Singh and Mishra [42], we have the subsequent theorem.

Theorem 3.10 Let $(\mathcal{L}, m_s)$ be a metric space and $\mathcal{M}$ an arbitrary non-empty set. Let $\mathcal{O}: \mathcal{M} \to CL(\mathcal{L})$ and $\mathcal{F}: \mathcal{M} \to \mathcal{L}$ with $\mathcal{F}(\mathcal{M})$ is superset of $\mathcal{O}(\mathcal{M})$, and $\mathcal{O}(\mathcal{M})$ or $\mathcal{F}(\mathcal{M})$ is CS of $\mathcal{L}$. Assume $\exists\, \alpha, \beta, \gamma \in [0, 1)$ and $\alpha + 2\beta + 2\gamma \in [0, 1)$ so that for every $d, e \in \mathcal{L}$,

$$\frac{1-\beta-\gamma}{1+\alpha} m_s(\mathcal{F}d, \mathcal{O}d) \leq m_s(\mathcal{F}d, \mathcal{F}e)$$

implies

$$\mathcal{H}(\mathcal{O}d, \mathcal{O}e) \leq a m_s(\mathcal{F}d, \mathcal{F}e) + \beta[m_s(\mathcal{F}d, \mathcal{O}d) + m_s(\mathcal{F}e, \mathcal{O}e)] + \gamma[m_s(\mathcal{F}d, \mathcal{O}e) + m_s(\mathcal{F}e, \mathcal{O}d)].$$

Then $\mathcal{F}$ and $\mathcal{O}$ have a CP, that is, there exists $r \in \mathcal{M}$ such that $fr = \mathcal{O}r$.

Further, $\mathcal{F}$ and $\mathcal{O}$ are ITC (just) at r and $\mathcal{F}r$ is FP of $\mathcal{F}$, if $\mathcal{M} = \mathcal{L}$, then $\mathcal{F}$ and $\mathcal{O}$ have a CFP.

Moreover, Theorem 2.7 and Theorem 3.1 was generalized in the following way by Singh and Mishra [41].

Theorem 3.11 Let $(\mathcal{L}, m_s)$ be a MS and $\mathcal{M}$ an arbitrary non-empty set. Let $\mathcal{O}: \mathcal{M} \to CL(\mathcal{L})$ and $\mathcal{F}: \mathcal{M} \to \mathcal{L}$ with $\mathcal{F}(\mathcal{M})$ is superset of $\mathcal{O}(\mathcal{M})$, and $\mathcal{O}(\mathcal{M})$ or $\mathcal{F}(\mathcal{M})$ is CS of $\mathcal{L}$. Assume $\exists\, a \in [0, 1)$ so that for every $d, c \in \mathcal{L}$,

$$\xi(a)\, m_s(\mathcal{F}d, \mathcal{O}d) \leq m_s(\mathcal{F}d, \mathcal{F}e)$$

implies

$$\mathcal{H}(\mathcal{O}d, \mathcal{O}e) \leq a\max\left\{m_s(\mathcal{F}d, \mathcal{F}e),\ \frac{m_s(\mathcal{F}d, \mathcal{O}d) + m_s(\mathcal{F}e, \mathcal{O}e)}{2},\ \frac{m_s(\mathcal{F}d, \mathcal{O}e) + m_s(\mathcal{F}e, \mathcal{O}d)}{2}\right\}.$$

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Then $\mathcal{F}$ and $\mathcal{O}$ have a CP, that is, there exists $r \in \mathcal{M}$ such that $fr = \mathcal{O}r$.

Further, if $\mathcal{M} = \mathcal{L}$, then $\mathcal{F}$ and $\mathcal{O}$ have a CFP provided that $\mathcal{F}$ and $\mathcal{K}$ are ITC (just) at r and $\mathcal{F}r$ is FP of $\mathcal{F}$.

The greater generality of Theorem 3.11 in comparison to Theorem 3.9 can be seen in the example that follows.

Example 3.1 Let $\mathcal{L} = \{1, 2, 3\}$ with the usual metric and take $\mathcal{K}$ and $\mathcal{F}$ as

$$\mathcal{O}\mathfrak{b} = \begin{cases} 2, 3 & \text{if } x \neq 3 \\ 3 & \text{if } x = 3, \end{cases} \text{and} \mathcal{F}\mathfrak{b} = \begin{cases} 1 & \text{if } x \neq 1 \\ 3 & \text{if } x = 1. \end{cases}$$

Then $\mathcal{O}$ does not satisfy Theorem 2.10, Indeed, for $b = 2$, $c = 3$,

$$m_s(2, \mathcal{O}2) = 0 \leq (1+ a)\, m_s(2, 3) = (1+ a)$$

and this does not reflect

$$1 = \mathcal{H}(\mathcal{O}2, \mathcal{O}3) \leq a m_s(2, 3) = a.$$

And, verification reveals that the pair of maps $\mathcal{O}$, $\mathcal{F}$ fulfills every assumptions of Theorem 3.11 and has a CP $r = 1$.

Furthermore, for a pair of maps, Kamal et al. [20] generalized Theorems 3.1 and 3.4 in the following way.

Theorem 3.12 Let $\mathcal{L}$ be a CMS and let $\mathcal{J}$ and $:\mathcal{L} \to CL(\mathcal{L})$. Assume that $\exists\, a \in [0, 1)$ so that for every d, $c \in \mathcal{L}$,

$$\Omega(a) \min\{m_s(d, \mathcal{J}d), m_s(e, \mathcal{O}e)\} \leq m_s(d, e) \text{ implies}$$

$$\mathcal{H}(\mathcal{J}d, \mathcal{O}e) \leq a \max\left\{m_s(d, e), \frac{m_s(d, \mathcal{J}d) + m_s(e, \mathcal{O}e)}{2}, \frac{m_s(d, \mathcal{J}e) + m_s(e, \mathcal{O}d)}{2}\right\}.$$

Then $\exists$ an element $r \in \mathcal{L}$ such that $r \in \mathcal{J}r \cap \mathcal{O}r$.

Theorem 3.8 of Dorić and Lazović and Theorem 3.12 of Kamal et al. [20] are generalized in the following theorem, which is attributed to Singh et al. [40].

Theorem 3.13 Let $\mathcal{L}$ be a CMS and let $\mathcal{J}$, $\mathcal{O}$: $\mathcal{L} \to CL(\mathcal{L})$. Assume that $\exists\, a \in [0, 1)$ so that for every b, $c \in \mathcal{L}$,

$$\chi(r) \min\{m_s(d, \mathcal{J}d), m_s(e, \mathcal{O}e)\} \leq m_s(d, e)$$

implies

$$\mathcal{H}(\mathcal{J}d, \mathcal{O}e) \leq a \max\left\{m_s(d, e), m_s(d, \mathrm{J}d), m_s(e, \mathrm{O}e), \frac{m_s(d, \mathcal{J}e) + m_s(e, \mathcal{O}d)}{2}\right\}.$$

Then $\exists$ an element $r \in \mathcal{L}$ with $r \in \mathcal{J}r \cap \mathcal{O}r$.

The generality of Theorem 3.13 is can be seen in the example given below.

Example 3.2 Consider $\mathcal{L} = \{(1, 1), (1, 4), (4, 1), (1,5), (5, 1), (4, 5), (5, 4)\}$ be endowed with the metric m_s defined by

$$m_s[(d_1, d_2), (e_1, e_2)] = /d_2 - e/ + /d_1 - e_1/.$$

Let $\mathcal{J}$ and $\mathcal{O}$ be such that

$$\mathcal{J}(d_1, d_2) = \begin{cases} (d_1, 1) & \text{if } d_1 \leq d_2 \\ (1, 1) & \text{if } d_1 > d_2 \end{cases} \text{ and} \mathcal{O}(\mathfrak{b}_1, \mathfrak{b}_2) = \begin{cases} (d_2, 1) & \text{if } d_1 \leq d_2 \\ (1, d_2) & \text{if } d_1 > d_2 \end{cases}.$$

It is easily confirmed that, for the maps $\mathcal{J}$ and $\mathcal{O}$, all the hypotheses of Theorem 3.13 are satisfied.

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Using a multivalued and two single valued maps, Singh et al. [45] obtained the CP and CFP theorem for a class of Ciric-Suzuki hybrid contraction in the manner that follows:

Theorem 3.14 Let $\mathcal{O}$: $\mathcal{M} \rightarrow$ CL($\mathcal{L}$) and $\mathcal{F}$, $\mathcal{G}$: $\mathcal{M} \rightarrow \mathcal{L}$. Asssume $\exists\, a \in [0,1)$ so that for all

$$d,\ e \in \mathcal{M},$$

$$\chi(a)\min\{m_s(\mathcal{F}d, \mathcal{O}d), m_s(\mathcal{G}e, \mathcal{O}e)\} \le m_s(\mathcal{F}d, \mathcal{G}e) \text{ implies}$$

$$\mathcal{H}(\mathcal{O}d, \mathcal{O}e) \le a\max\left\{m_s(\mathcal{F}d, \mathcal{G}e),\ m_s(Fd, \mathrm{O}d), m_s(\mathcal{G}e, \mathrm{O}e),\ \frac{m_s(\mathcal{G}d, \mathcal{O}e) + m_s(Fe,\ \mathcal{O}d)}{2}\right\}.$$

If anyone of the following is a CS of $\mathcal{L}$: $\mathcal{O}(\mathcal{M})$, $\mathcal{F}(\mathcal{M})$, or $\mathcal{G}(\mathcal{M})$,

then

(i) The pair of maps, ($\mathcal{O}$, $\mathcal{F}$) is nonempty; that is, $\exists$a point $r \in \mathcal{M}$ such that $\mathcal{F}r \in \mathcal{O}r$.

(ii) The pair of maps, ($\mathcal{O}$, $\mathcal{G}$) is nonempty; that is, $\exists$a point $r_1 \in \mathcal{M}$ such that $\mathcal{G}r_1 \in \mathcal{O}r_1$. Further if, $\mathcal{M}=\mathcal{L}$, then

(iii) If the maps $\mathcal{F}$ and $\mathcal{O}$ are ITC just at CP r and $\mathcal{F}r$ is FP of $\mathcal{F}$, then $\mathcal{F}$ and $\mathcal{O}$ have a CFP.

(iv) If the maps g and $\mathcal{O}$ are ITC just at CP r_1 and $\mathcal{G}r_1$ is FP of $\mathcal{G}$, then $\mathcal{G}$ and $\mathcal{O}$ have a CFP.

(v) As long as (iii) and (iv) are true, the maps $\mathcal{O}$, $\mathcal{F}$, and $\mathcal{G}$ have a CFP.

The following novel result was obtained by R.K. Bose [7]:

Theorem 3.15 Let $\mathcal{L}$ be a CMS and let $\mathcal{J}$, $\mathcal{O}$: $\mathcal{L} \rightarrow$ CB($\mathcal{L}$). Assume $\exists\alpha, \beta, \gamma, \delta, \theta \in [0, 1)$ with $\alpha + \beta + \gamma + \delta + \theta < 1$ and, with $\beta = \gamma$ or $\delta = \theta$ so that

$$w\min\{m_s(d, \mathcal{J}d),\ m_s(e, \mathcal{O}e)\} \le m_s(d, e) \text{ implies}$$

$$\mathcal{H}(\mathcal{J}d, \mathcal{O}e) \le \alpha\, m_s(d, e) + \beta\, m_s(d, \mathcal{J}d) + \gamma\, m_s(e, \mathcal{O}e) + \delta m_s(d, \mathcal{J}e) + \theta\, m_s(e, \mathcal{O}d)$$

for all d, $e \in \mathcal{L}$, and $w_1 = \frac{1-\alpha-\gamma}{1-\gamma+\delta+\theta}$, $w_2 = \frac{1-\beta-\delta}{1+\gamma-\delta+\theta}$ and $w = \min\{w_1, w_2\}$.

Then $\exists$an element $r \in \mathcal{L}$ so that $r \in \mathcal{J}r \cap \mathcal{O}r$.

The following result for two multivalued maps and a single valued map is due to Singh et al. [44].

Theorem 3.16 Let $\mathcal{J}$, $\mathcal{O}$: $\mathcal{M} \rightarrow$ CL($\mathcal{L}$) and let $\mathcal{F}$, $\mathcal{G}$: $\mathcal{M} \rightarrow \mathcal{L}$. Assume $\exists\, a \in [0,1)$ so that for all

$$d,\ e \in \mathcal{M},$$

$$\chi(a)\min\{m_s(\mathcal{F}d, \mathcal{J}d), m_s(\mathcal{G}e, \mathcal{O}e)\} \le m_s(\mathcal{F}d, \mathcal{G}e) \text{ implies}$$

$$\mathcal{H}(\mathcal{J}d, \mathcal{O}e) \le a\max\left\{m_s(\mathcal{F}d, \mathcal{G}e),\ m_s(Fd, \mathrm{J}d), m_s(\mathcal{G}e, \mathrm{O}e),\ \frac{m_s(\mathcal{F}d, \mathcal{J}e) + m_s(\mathcal{G}e,\ \mathcal{O}d)}{2}\right\}.$$

If anyone of the following is a CS of $\mathcal{L}$: $\mathcal{O}(\mathcal{M})$, $\mathcal{F}(\mathcal{M})$, or $\mathcal{G}(\mathcal{M})$,

then

(i) The pair of maps, ($\mathcal{J}$, $\mathcal{F}$) is nonempty; that is, $\exists$a point $r \in \mathcal{M}$ such that $\mathcal{F}r \in \mathcal{J}r$.

(ii) The pair of maps, ($\mathcal{O}$, $\mathcal{G}$) is nonempty; that is, $\exists$a point $r_1 \in \mathcal{M}$ such that $\mathcal{G}r_1 \in \mathcal{K}r_1$. Further if, $\mathcal{M}=\mathcal{L}$, then

(iii) If the maps $\mathcal{J}$ and $\mathcal{F}$ are ITC just at CP r and $\mathcal{F}r$ is FP of $\mathcal{F}$, then $\mathcal{J}$ and $\mathcal{F}$ admit a CFP.

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(iv) If the maps $\mathcal{O}$ and g are ITC just at CP r_1 and $\mathcal{G}r_1$ is FP of $\mathcal{G}$, Then $\mathcal{O}$ and $\mathcal{G}$ admit a CFP.

(v) As long as (iii) and (iv) are true, then $\mathcal{J}$, $\mathcal{O}$, $\mathcal{F}$, and $\mathcal{G}$ have a CFP.

In 2013, Rao *et al.* [38] introduced a new contraction and proved a common coupled fixed point as follows:

Theorem 3.17 Let: $\mathcal{L}\times\mathcal{L}\to\mathcal{L}$ and $\mathcal{F}$: $\mathcal{L}\to\mathcal{L}$ with $\mathcal{K}(\mathcal{L}\times\mathcal{L})\subseteq\mathcal{F}(\mathcal{L})$ and f($\mathcal{L}$) is complete. If $\exists\, a\in [0, 1)$ so that

$\Omega(a)m_s(\mathcal{F}d, (d, e)) \leq \max\{m_s(\mathcal{F}d, \mathcal{F}p), m_s(\mathcal{F}e, \mathcal{F}q), m_s(\mathcal{F}d, \mathcal{O}(d, e)), m_s(\mathcal{F}e, \mathcal{O}(e, d))\}$ implies

$m_s((d, e), \mathcal{O}(p, q)) \leq a\max\{m_s(\mathcal{F}d, \mathcal{F}p), m_s(\mathcal{F}e, \mathcal{F}q), m_s(\mathcal{F}d, \mathcal{O}(d, e)), m_s(\mathcal{F}e, \mathcal{O}(e, d)), m_s(\mathcal{F}p, \mathcal{O}(p, q)), m_s(\mathcal{F}q, \mathcal{O}(q, p)), m_s(\mathcal{F}p, \mathcal{O}(d, e)), m_s(\mathcal{F}q, \mathcal{O}(e, d))\}$,

for all $d, e, p, q\in\mathcal{L}$.

Then $\mathcal{O}$ and $\mathcal{F}$ have a unique CdCFP provided that $\mathcal{O}$ and $\mathcal{F}$ are w-compatible.

IV. Fixed Point Theorems for Contractive Maps

The classical FP theorem of Edelstein [16], that is, Theorem 1.2 (above) is generalized as follows by Suzuki [48].

Theorem 4.1 Let $(\mathcal{L}, m_s)$ be a CoMS and let $\mathcal{O}$: $\mathcal{L} \to \mathcal{L}$ such that

$$\frac{1}{2}m_s(d, \mathcal{O}d) < m_s(d, e) \text{ implies } m_s(\mathcal{O}d, \mathcal{O}e) < m_s(d, e), \text{ for all } d, c \in \mathcal{L}.$$

Then $\mathcal{O}$ has a unique FP.

Suzuki [47] also obtains a generalization of the Meir-Keeler FP theorem [29], viz. Theorem 1.4 (above).

Theorem 4.2 Let $(\mathcal{L}, m_s)$ be a CMS and let :$\mathcal{L}\to\mathcal{L}$ fulfilling the requirement:

for a given $\varepsilon > 0, \exists\delta > 0$ such that for every $b, c \in \mathcal{L}$,

(i) $m_s(d, e) < \varepsilon + \delta$ and $\frac{1}{2}m_s(d, \mathcal{O}d) < m_s(d, e)$ implies $m_s(\mathcal{O}d, \mathcal{O}e) \leq \varepsilon$;

(ii) $\frac{1}{2}m_s(d, \mathcal{O}d) < m_s(d, e)$ implies $m_s(\mathcal{O}d, \mathcal{O}e) < m_s(d, e)$.

Then $\mathcal{O}$ has a unique FPin $\mathcal{L}$.

Kikkawa and Suzuki [26] obtained a novel result in this direction for a pair of maps that generalize Theorem 4.2 as follows.

Theorem 4.3 Let $(\mathcal{L}, m_s)$ be a CMS and let $\mathcal{F}, \mathcal{O}$: $\mathcal{L}\to\mathcal{L}$ with $(\mathcal{L}) \subset \mathcal{F}(\mathcal{L})$ and $\mathcal{F}$ is continuous:

Also, assume for a given $\varepsilon > 0$, $\exists\delta > 0$ such that, for all $b, c\in\mathcal{L}$,

(i) $m_s(\mathcal{F}d, \mathcal{F}e) < \varepsilon + \delta$ and $\frac{1}{2}m_s(\mathcal{F}d, \mathcal{O}d) < m_s(\mathcal{F}d, \mathcal{F}e)$ implies $m_s(\mathcal{O}d, \mathcal{O}e) \leq \varepsilon$;

(ii) $\frac{1}{2}m_s(\mathcal{F}d, \mathcal{O}d) < m_s(\mathcal{F}d, \mathcal{F}e)$ implies $m_s(\mathcal{O}d, \mathcal{O}e) < m_s(\mathcal{F}d, \mathcal{F}e)$.

Then $\exists$a unique CFP of $\mathcal{F}$ and $\mathcal{O}$ provided both commute at every point of $\mathcal{L}$.

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V. Fixed Point Theorems for Interpolative Contraction Maps

The celebrated Kannan theorem was recently reexamined by taking the interpolation theory into consideration [23]. For a metric space ($\mathcal{L}$, d), the self-mapping $:\mathcal{L}\to\mathcal{L}$ is said to be an Kannan type interpolative contraction (KTIC), if there are scalars$\lambda\in[0,1)$ and $\kappa\in(0,1)$ such that

$$m_s(\mathcal{O}d, \mathcal{O}c) \leq \lambda[m_s(d,\mathcal{O}d)]^{\kappa}[m_s(c,\mathcal{O}c)]^{1-\kappa} \text{ for each } d, c\in X \text{ with } d\neq\mathcal{O}d.$$

The next concept of Hardy-Rogers interpolative type contraction was first presented by Karapiner et al. [24].

Theorem 5.1 Let ($\mathcal{L}$, m_s) be a CMS and the self-mapping $\mathcal{O}$: $\mathcal{L}\to\mathcal{L}$. If ∃constants $\lambda\in[0,1)$ and $\kappa\in(0,1)$ so that

$$\frac{1}{2}\, m_s(d, \mathcal{O}d) \leq m_s(d, e) \text{ implies}$$

$$w(d, c)m_s(\mathcal{O}d, \mathcal{O}e) \leq \varphi[m_s(d,e)]^{\alpha}[m_s(d,\mathcal{O}d)]^{\beta}[m_s(e,\mathcal{O}e)]^{1-\gamma}\frac{1}{2}\left[m_s(d,\mathcal{O}e)+m_s(e,\mathcal{O}d)\right]^{1-\alpha-\beta-\gamma})$$

for all $d, e\in\mathcal{L}\backslash\text{Fix}(\mathcal{O})$.

Then $\mathcal{O}$ has a FP in $\mathcal{L}$ if $\mathcal{O}$ is continuous, w-orbital admissible $w(d_0, \mathcal{O}d_0)\geq 1$ for some $d_0\in\mathcal{L}$ and $\mathcal{O}^2$ is continuous with $w(d, \mathcal{O}d)\geq 1$ where $d\in \text{Fix}(\mathcal{O}^2)$

Interpolative Kannan-Meir-Keeler type contraction (IKMKC) was first defined by Karapiner [22].

The self-mapping $\mathcal{O}$: $\mathcal{L}\to\mathcal{L}$is called an IKMKC if there are scalars$\gamma\in[0,1)$ so that

For every $d, e\in\mathcal{L}\backslash\text{Fix}(\mathcal{O})$, Given $\varepsilon>0$, ∃$\delta>0$ such that $\varepsilon<[m_s(d,\mathcal{O}d)]^{\beta}[m_s(e,\mathcal{O}e)]^{1-\beta}<\varepsilon+\delta$ implies $m_s(\mathcal{O}d, \mathcal{O}e)\leq\varepsilon$; and $m_s(\mathcal{O}d, \mathcal{O}e)<[m_s(d,\mathcal{O}d)]^{\beta}[m_s(e,\mathcal{O}e)]^{1-\beta}$.

VI. Conclusion

The work presented in this paper projects an expository review of literature concerning the extensions and generalizations of Suzuki contraction theorem. One common challenge faced by researchers is the deficiency of comprehensive literature covering advancements of Suzuki contraction theorem. This paper progressively tackles this issue by providing a comprehensive resource that provides deep insights into the sequel of its advancements. Present work saves the time of researchers to survey literature from multiple sources.

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