Use a Combination of Two Methods: The Projection Plane Replacement Method and the Cone Method Circumscribing the Sphere to Solve Complex Geometry Problems

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Abstract - **Geometry is a complex science and is widely used in daily life. Descriptive geometry is widely used in design works and construction of technical drawings for manufacturing. Many complex problems cannot be solved without finding a suitable auxiliary configuration. This article proposes a method to solve complex problems. This method will combine 2 methods of replacing the projection plane and the method of the cone circumscribing the sphere to solve the following complex problems.**

Keywords: Sphere: sp, Cone: co, Plane: pl.

I. INTRODUCTION

1.1 Projection plane replacement method:

This method includes the following steps:

(See Fig 1) and (Fig 2)

- Construct a new projection plane
- Project the image onto the new projection plane to get a new projection
- Solve exercises on the new projection
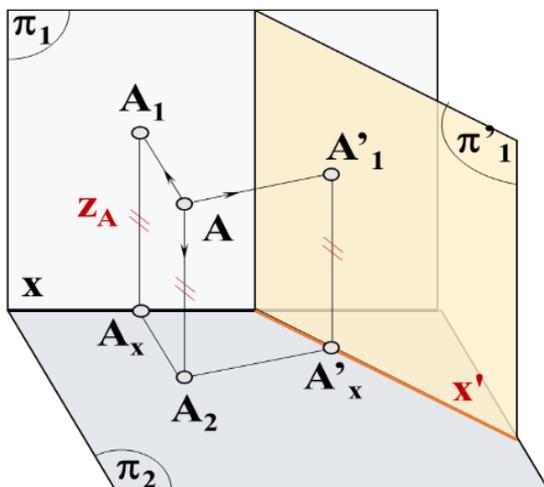
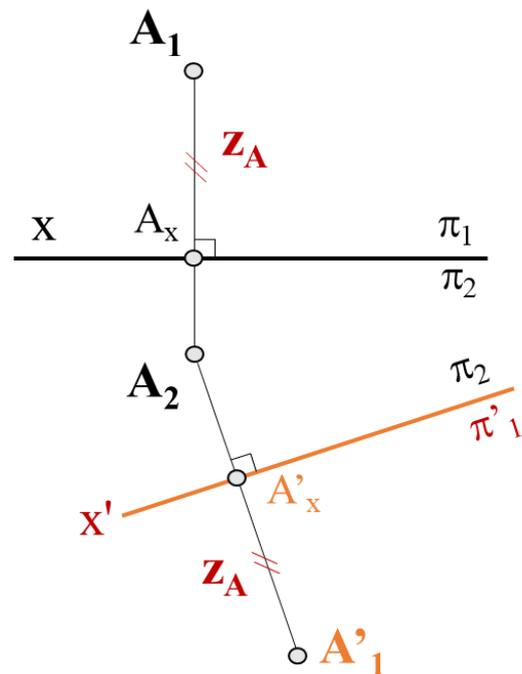*Example*: Replace plane π_1 with new plane π'_1

Figure 1

Figure 2

1.2 Method of a cone tangent to a sphere:

On the axis of the co, we take a point. Using this point as the center, construct a sp tangent to the co.

- If a pl is tangent to the co, it is also tangent to the sp. This common tangent pl will contain a generating line of the co. (see Fig.3. a).
- If a pl passes through the apex of the co and is tangent to the sp, then it is also tangent to the co. (see Fig.3. a).
- There are many sp inscribed in a co. They touch the co in circles. These circles can be taken as the base circle of the co. The centers of these sp lie on the axis of the co. (See Fig.3.b).
- There are many co circumscribing a sp. The axes of these co converge at the center of the sp. (see Fig.3.c).
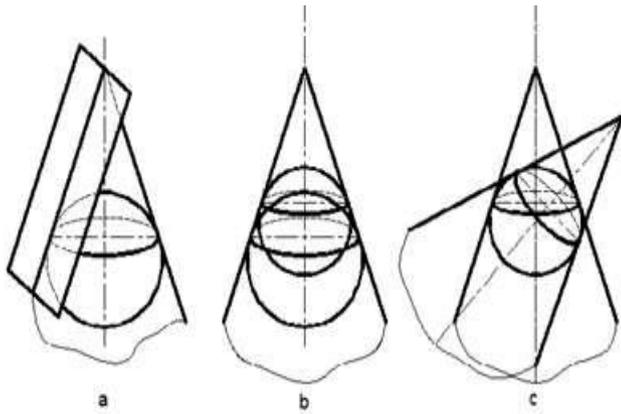
Figure 3

II. APPPLICATION TO SOLVE SOME GEOMETRIC PROBLEMS

2.1 Problem 1

Draw the end generators of a right circular co with a as the axis of revolution and the vertex angle as 2α. Point Sis the vertex.
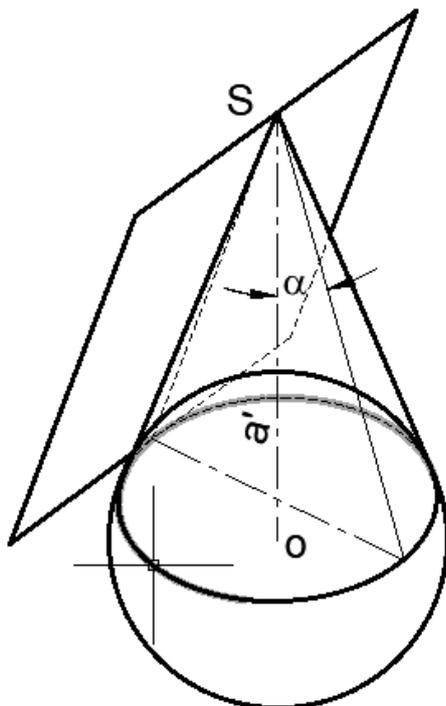
(See Figure 4)

Figure 4

Solution: (See Figure 5)

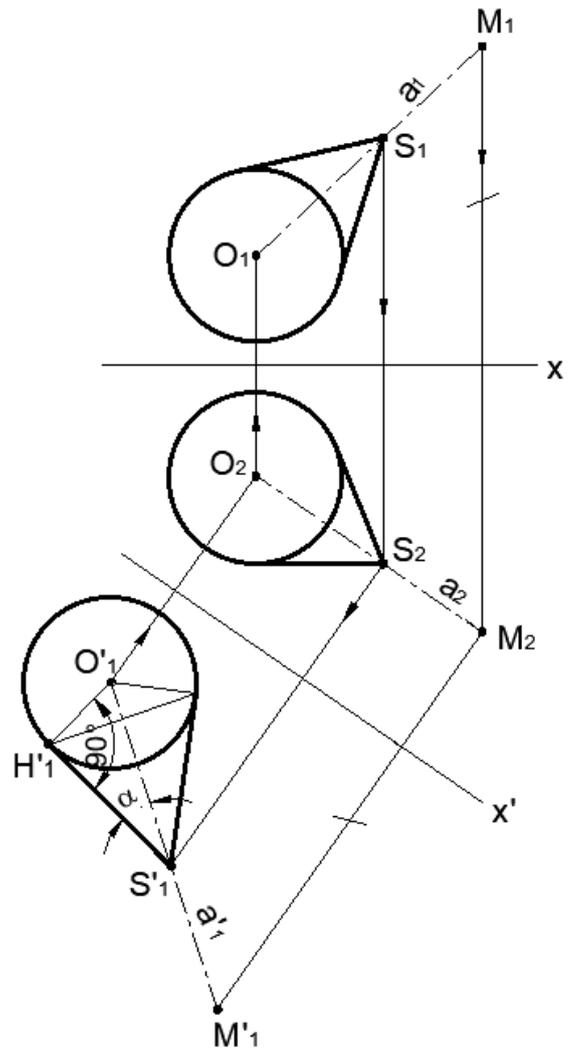- Replace plane π_1 with new plane π'_1

Figure 5

- Draw line x' parallel to line a_2. In the new projection plane system, a is the front line.
- Draw a line S'1H'1 that makes an angle of α with line a'1. Point H'1 is taken at any point.
- Draw a line through H'1 and perpendicular to S'1H'1 that intersects a'1 at point O'1
- Construct a sphere with center O passing through H.
- SH is the end generator of the cone.
- Draw the remaining generator tangent to the sphere.

2.2 Problem 2

Given two skew lines a and b. Construct a plane P satisfying the following two constraints:

- The angle between P and line a is α
- The angle between P and line b is β

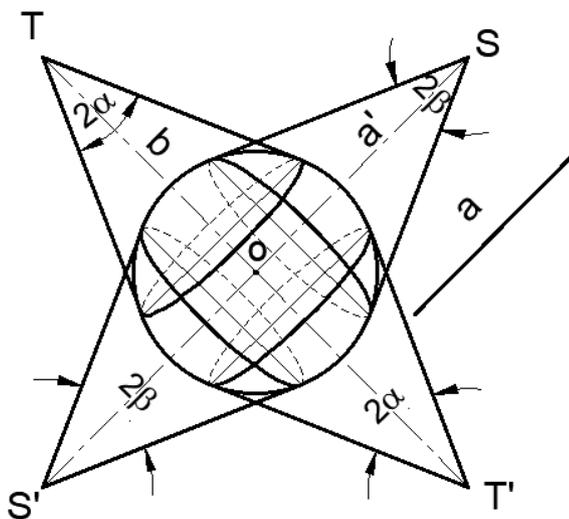**Solution:** (See Figure 6 and Figure 7)

Figure 6

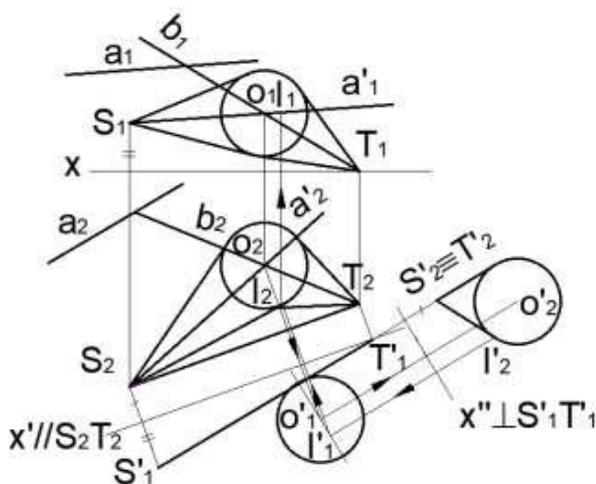Based on section 2.1 we will find points S and T. Then we proceed to solve as follows:

Figure 7

- Take any point O on line b
- Construct line a' passing through point O and parallel to line a
- Construct a sphere with center O and any radius
- Construct two cones with vertices S and T tangent to the sphere. These two cones have axes as 2 lines a' and b. These two cones have vertex angles of 2α and 2β.
- The Plane P containing line ST tangent to the sphere is the plane to be constructed. Plane P is tangent to the sphere so it is also tangent to both cones.
- In this exercise, we have to replace the projection plane twice to convert line ST into a projection line.

Conclusion

Thus, there will be 4 lines ST, S'T', ST' and TS'. So this problem will have a total of 8 geometric solutions. Here the author only draws one geometric solution, which is the plane IST.

2.3 Problem 3

Given line d and 2 points A, B not on d.

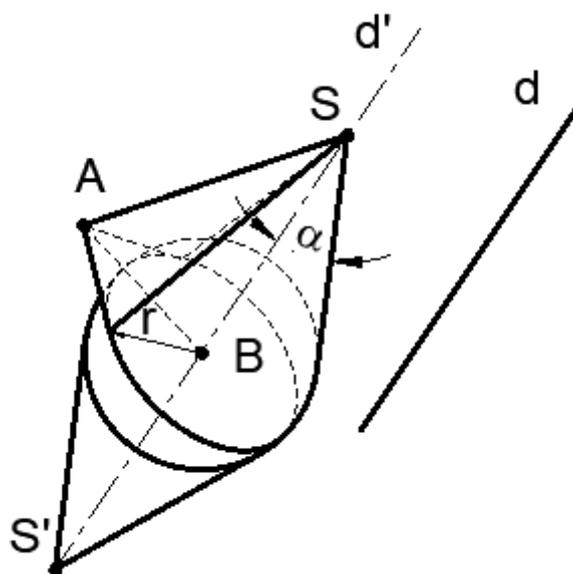Construct plane P at a distance r from B and make an angle α with d (See Figure 8)

Figure 8

Solution: (See Figure 8 and Figure 9)

- Draw line d' passing through point B and parallel to line d.
- Construct a sphere with center B and radius r
- Construct a right circular cone with axis d' and a vertex angle equal to 2α.
- Based on problem 2.1 to determine vertex S of the cone.
- Construct a plane passing through SA and tangent to the sphere. This plane is also tangent to the cone

The problem has 4 solutions and has 2 cones with vertices S and S'. Here the author only draws two geometric solutions, which are the plane SAH and the plane SAG.

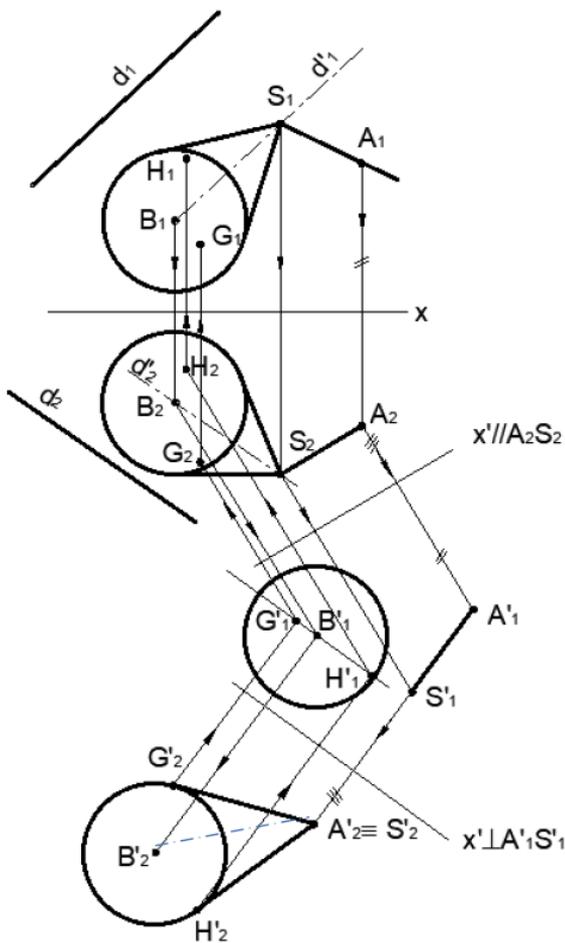To solve this problem, we have to replace the projection plane twice.

Figure 9

III. CONCLUSION

Geometry is a science that is widely applied in engineering and life. It is very complicated and there are many ways to solve it. To solve geometric problems, we must think for ourselves and find solutions. So writing articles about this subject is very difficult. Because it has no experiments, there are no cited experimental data. This article presents a new solution to solve some geometric problems. It helps us solve faster and easier to understand.

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